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Extended Thomas-Fermi estimates of single particle potentials for doubly magic and superheavy nuclei

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ABSTRACT

We present self-consistent semiclassical calculations using Skyrme type effective interactions to determine local quantities such as nuclear densities and single particle potentials. These local quantities are given in an analytical form depending only on a few parameters characterizing the neutron and proton densities of nuclei from ^{16}O to the region of superheavy nuclei. These parameters are given as a function of the mass number A and the isospin parameter $I = (N - Z)/A$.

1. INTRODUCTION

The Extended Thomas-Fermi (ETF) method [1, 2] which determines the global stucture of nuclei in a self-consistent way by a density variational calculation [3], particularly in connection with effective interactions of the Skyrme type, has proven extremely efficient over the last two decades in a precise description of average nuclear properties.

In this work we are not so much interested in nuclear binding energies, even though it is well known that the ETF approach is very performant in this field yielding binding energies very close to the expermental data [4, 1], as long as a reasonable effective interaction is used. The aim of the present work is rather to concentrate on local properties such as density distribution and mean field potentials for protons or neutrons and to show how these quantities can be obtained in analytical form for any nucleus throughout the periodic table.

Using the ETF approach we determine, as the result of the density variational procedure, self-consistent neutron and proton densities for a very wide sample of nuclei, ranging from ^{16}O to the region of superheavy nuclei. These densities are obtained as modified Fermi functions, taking into account the asymmetry of the nuclear surface. Such a variation in a limitted variational space has been demonstrated to constitute a very good approximation and to yield energies and density distributions very close to those of the solution of the full Euler-Lagrange variational equation [5]. This is demonstrated in section 2.1.

The ETF method allows to express quantities like the kinetic energy density $\tau(\vec{r})$ and the spin-orbit density $\vec{J}(\vec{r})$ as functions of the local density ρ and its derivatives. For Skyrme forces where the total nuclear energy is given as functional of such densities $\rho_q(\vec{r})$, $\tau_q(\vec{r})$ and $\vec{J}_q(\vec{r})$, $q=\{n,p\}$, one is able to express all form factors, like the central potentals $V_n(\vec{r})$ and $V_p(\vec{r})$ (but also effective mass and spin-orbit potentials) as functions of the local densities $\rho_n(\vec{r})$ and $\rho_p(\vec{r})$ and their derivatives. As the nucleon densities are given in analytic form, this implies that all these form factors are given analytically. This will be explained explicitly in section 2.2.

To be able to use this procedure not only for a given nucleus but, in fact, for any nucleus in the periodic table, the parameters of the modified Fermi functions are analyzed for a vast sample of nuclei to obtain an analytic expression for them as a function of the mass number A and isospin parameter $I = (N-Z)/A$, allowing the interested reader to obtain not only neutron and proton densities but also single-particle potentials, effective masses and spin-orbit potentials in an analytical form and this for any nucleus throughout the periodic table. To test the dependence of our analysis on the effective nucleon-nucleon force used, which constitutes the only input to our method, this procedure is carried out for three of the most sucessfull Skyrme interactions: SIII [6], SkM* [4] and the SLy4 force [7]. The parameters determining each of these interactions are given in Table 1.

Whereas the density dependence of the effective mass, the Coulomb and spin-orbit potential is rather trivial, the one of the nuclear central potential is more complicated, as will be seen in section 2.2 below. For this reason, we thought that a still easier analytical description of all these Skyrme central potentials seems in place for the reader very much used to single-particle potentials of the Woods-Saxon type [8]. We have therefore made some least-square adjustment of the Skyrme-ETF forme factors as explained in detail in section 3.

We have limited our investigations to spherical nuclei. The reason is that the ETF method is a semiclassical approach, i.e. by nature of the type of the liquid-drop model, where ground states are always spherical. Once the structure for a given nucleus at *spherical* symmetry is established, one

Table 1. Skyrme forces

Parameter	SkM*	SkIII	SLy4
t_0[MeV· fm^3]	-2645	-1128.75	-2488.9
t_1[MeV· fm^5]	410	395	486.82
t_2[MeV· fm^5]	-135	-95	-546.3
t_3[MeV· fm^4]	15595	14000	13777
x_0	0.09	0.45	0.834
x_1	0	0	-0.344
x_2	0	0	-1
x_3	0	1	1.354
W_0[MeV· fm^5]	120	120	123
α	0.167	1	0.167

can easily deform the nuclear shape using some convenient paramatrization of the nuclear deformation (see [9]).

As explained shell oscillations are, by definition, absent from our semi-classical description. However, they can be included by the Strutinsky method, as formulated e.g. in ref. [10].

One could finally object that nowadays Hartree-Fock calculations have become so performant that the use of any semiclassical approach which corresponds only to an approximation thereof seem inadequate. Such an argument holds as long as one is considering a single nucleus at a given deformation. However, if one is interested in a more systematic study of the potential energy surfaces and over a whole range of nuclei it is evident that the ETF approach, even including a shell correction calculation, is at least factor of 10 faster that the corresponding Hartree-Fock approach. This is even more the case with the method presented here, which determines the ETF density and potential parameters as a function of mass number and isospin dependence. One could even argue that our Skyrme ETF form factors constitute an ideal and easy to use initial guess for performing HF calculations.

We have performed our calculations for an ensemble of doubly magic nuclei which are known to have spherical symmetry (^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{132}Sn, ^{80}Zn, ^{90}Zn, ^{132}Ce, ^{208}Pb) and for 9 hypotetical (yet Unknown: Uk) superheavy nuclei (SHN) ($^{254}_{110}$Uk, $^{264}_{110}$Uk, $^{274}_{110}$Uk, $^{272}_{118}$Uk, $^{282}_{118}$Uk, $^{292}_{118}$Uk, $^{290}_{126}$Uk, $^{300}_{126}$Uk, $^{310}_{126}$Uk). The hypothetical superheavy nuclei have been chosen in such a way as to probe both their A and isospin dependence.

2. EXTENDED THOMAS-FERMI MODEL

2.1. DENSITY VARIATIONAL CALCULATIONS

The reason why we use effective interactions of the Skyrme type consists in the fact that for such nuclear forces the total energy of the nuclus can be written as a functional of the local density $\rho_q(r)$, the kinetic energy densities $\tau_q(r)$ and the spin-orbit densities $\vec{J}_q(r)$, where q stands for either of the two kinds of nucleons q=$\{n, p\}$:

$$E[\rho_p, \rho_n] = \int \mathcal{E}[\rho_q(\vec{r}), \tau_q(\vec{r}), \vec{J}_q(\vec{r})] d^3r = 0\,, \tag{1}$$

where $\mathcal{E}[\rho_q(\vec{r}), \tau_q(\vec{r}), \vec{J}_q(\vec{r})]$ is the total energy density. One of the properties of the ETF approach consists in the fact that it allows for a semiclassical expansion of the kinetic energy density $\tau_q(\vec{r})$ and of the spin-orbit density $\vec{J}_q(r)$ as functions of the local density $\rho_q(r)$ and of its derivatives. This semiclassical expansion in powers of $\hbar$ has the following form for the kinetic energy density

$$\tau[\rho] = \tau^{(TF)}[\rho] + \tau^{(2)}[\rho] + \tau^{(4)}[\rho]\,, \tag{2}$$

where $\tau^{(TF)}[\rho]$ is the Thomas-Fermi term given by

$$\tau_q^{(TF)}[\rho_q] = \frac{3}{5}(3\pi^2)^{3/2}\,\rho_q^{5/3}(\vec{r})\,, \tag{3}$$

and $\tau^{(2)}[\rho]$ and $\tau^{(4)}[\rho]$ are the semiclassical corrections of order $\hbar^2$ and $\hbar^4$ respectively. The second order term $\tau^{(2)}[\rho]$ is given by

$$\begin{aligned} \tau_q^{(2)}[\rho_q] \;\; = \;\; & \frac{1}{36}\frac{(\vec{\nabla}\rho_q)^2}{\rho_q} + \frac{1}{3}\Delta\rho_q + \frac{1}{6}\frac{\vec{\nabla}\rho_q \cdot \vec{\nabla} f_q}{f_q} + \frac{1}{6}\rho_q\frac{\Delta f_q}{f_q} \\ & - \frac{1}{12}\rho_q\left(\frac{\vec{\nabla} f_q}{f_q}\right)^2 + \frac{1}{2}\left(\frac{2m}{\hbar^2}\right)^2 \rho_q \left(\frac{\vec{W}_q}{f_q}\right)^2 . \end{aligned} \tag{4}$$

where

$$f_q(\vec{r}) = \frac{m}{m^*(\vec{r})}\,, \tag{5}$$

is the effective-mass form factor and where $\vec{W}_q(\vec{r})$ is the spin-orbit potential. The fourth-order term $\tau_q^{(4)}[\rho]$ is rather lengthy and will not be given here but can be found in [11, 12] where it is also shown that its contribution to integrated quantities such as the total nuclear energy has to be taken into account but that it can be neglected alltogether in the calculation of nuclear potentials.

The spin being a pure quantum phenomenum, the spin-orbit density $\vec{J}^{(TF)}$ at the Thomas-Fermi level has to be zero and the lowest-order non trivial contribution in the semiclassical expansion is of order $\hbar^2$:

$$\vec{J}_q[\rho] = \vec{J}_q^{(2)}[\rho] + \vec{J}_q^{(4)}[\rho] \ . \tag{6}$$

where

$$\vec{J}_q^{(2)}[\rho] = -\frac{2m}{\hbar^2}\frac{\rho_q}{f_q}\vec{W}_q \,, \tag{7}$$

and where $\vec{J}_q^{(4)}[\rho]$ can be found in [12].

One now is able to express the total energy in eq. (1) as a functional of the local densities $\rho_n(r)$ and $\rho_p(r)$ alone. As a consequence, one can perform density variational calculations where the variational quantities are not wavefunctions as is the case in the Hartree-Fock approach but rather the neutron and proton densities. As the functional expressions for the kinetic energy density $\tau[\rho]$ and for the spin-orbit density $\vec{J}[\rho]$ are of semiclassical origin, it only makes sense to use densities of the liquid-drop type (i.e. without shell oscillations) in these functional expressions. It has been shown [5, 1] that the densities obtained by solving the Euler-Lagrange equations which follow from the variational procedure are, indeed, very close to the so-called *modified Fermi functions*

$$\rho(r) = \frac{\rho_c}{(1 + \exp\frac{r - R^\rho}{a^\rho})^\gamma} \,, \tag{8}$$

where the parameter γ allows for an asymmetry of the density distribution in the nuclear surface. When one performs a density-variational calculation in the restricted subspace of these modified Fermi functions, the variational parameters are the coefficients ρ_c, R^ρ, a^ρ and γ for each kind of nucleons, protons and neutrons. Out of these eight variational parameters two are determined by the constraint on the particle numbers Z and N.

2.2. SKYRME-FORCE FORM FACTORS

One starts from the energy density for a Skyrme force [13]

$$\begin{aligned}\mathcal{E}[\rho_q(\vec{r}), \tau_q(\vec{r}), \vec{J}_q(\vec{r})] \quad = \quad & \frac{\hbar^2}{2m}\tau + \frac{1}{2}t_0[(1 + \frac{x_0}{2})\rho^2 - (x_0 + \frac{1}{2})(\rho_n^2 + \rho_p^2)] \\ & + \frac{1}{12}t_3\rho^\alpha[(1 + \frac{x_3}{2})\rho^2 - (x_3 + \frac{1}{2})(\rho_n^2 + \rho_p^2)] \\ & + \frac{1}{4}[t_1(1 + \frac{x_1}{2}) + t_2(1 + \frac{x_2}{2})]\tau\rho\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{4}[-t_1(x_1+\frac{1}{2})+t_2(x_2+\frac{1}{2})](\tau_n\rho_n+\tau_p\rho_p)\\
&+\frac{1}{16}[3t_1(1+\frac{x_1}{2})-t_2(1+\frac{x_2}{2})](\vec\nabla\rho)^2\\
&-\frac{1}{16}[3t_1(x_1+\frac{1}{2})+t_2(x_2+\frac{1}{2})][(\nabla\rho_n)^2+(\vec\nabla\rho_p)^2]\\
&+\frac{W_0}{2}[\vec J\cdot\vec\nabla\rho+\vec J_n\cdot\vec\nabla\rho_n+\vec J_p\cdot\vec\nabla\rho_p]+\mathcal{E}_{\rm Coul} \qquad (9)
\end{aligned}
$$

where $\mathcal{E}_{\rm Coul}$ is the Coulomb energy density which can be written as the sum of a direct and an exchange contribution which we evaluate in the Slater approximation[14, 15]

$$
\mathcal{E}_{\rm Coul}(\vec r)=\frac{e^2}{2}\rho_p(\vec r)\int d^3r\,\frac{\rho_p(\vec{r}\,')}{|\,\vec r-\vec{r}\,'\,|}-\frac{3}{4}e^2(\frac{3}{\pi})^{1/3}\,\rho_p^{4/3}(\vec r)\;. \qquad (10)
$$

Applying the variational principle which states that the total energy of eq. (1) should be stationary with respect to any variation of the single-particle wavefunctions $\varphi_j^{(q)}$ one obtains for Skyrme type effective interactions the Skyrme HF equation

$$
\hat{\mathcal{H}}_q=\left(-\vec\nabla\frac{\hbar^2}{2m_q^*(\vec r)}\vec\nabla+V_q(\vec r)-i\vec W_q(\vec r)\cdot(\vec\nabla\times\vec\sigma)\right)\varphi_j^{(q)}=\varepsilon_j^{(q)}\varphi_j^{(q)}\,. \qquad (11)
$$

The central nuclear potential $V_q(\vec r)$ is obained as a funtional derivative of the energy density, eq. (9):

$$
V_q(\vec r)=\frac{\delta\mathcal{E}(\vec r)}{\delta\rho_q(\vec r)}=\frac{\partial\mathcal{E}}{\partial\rho_q}-\vec\nabla\left(\frac{\partial\mathcal{E}}{\partial(\vec\nabla\rho_q)}\right)+\Delta\left(\frac{\partial\mathcal{E}}{\partial(\Delta\rho_q)}\right)\,. \qquad (12)
$$

which for the Skyrme forces yields

$$
\begin{aligned}
V_q(\vec r) \;=\;& t_0(1+\frac{x_0}{2})\rho-t_0(x_0+\frac{1}{2})\rho_q\\
&+\frac{t_3}{12}\alpha\rho^{\alpha-1}[(1+\frac{x_3}{2})\rho^2-(x_3+\frac{1}{2})(\rho_n^2+\rho_p^2)]\\
&+\frac{t_3}{6}\rho^{\alpha}[(1+\frac{x_3}{2})\rho-(x_3+\frac{1}{2})\rho_q]\\
&+\frac{1}{4}[t_1(1+\frac{x_1}{2})+t_2(1+\frac{x_2}{2})]\tau+\frac{1}{4}[-t_1(x_1+\frac{1}{2})+t_2(x_2+\frac{1}{2})]\tau_q\\
&-\frac{1}{8}[3t_1(1+\frac{x_1}{2})-t_2(1+\frac{x_2}{2})](\vec\nabla^2\rho)\\
&+\frac{1}{8}[3t_1(x_1+\frac{1}{2})+t_2(x_2+\frac{1}{2})](\vec\nabla^2\rho_q)\\
&-\frac{W_0}{2}[\,{\rm div}\vec J+\,{\rm div}\vec J_q]+V_{\rm Coul} \qquad (13)
\end{aligned}
$$

where the Coulomb potential is given by

$$V_{\rm Coul}(\vec{r}) = e \int \frac{\rho_p(\vec{r}')}{|\,\vec{r} - \vec{r}'\,|} d^3r' - e(\frac{3}{\pi})^{1/3} \rho_p^{1/3}(\vec{r}) \; . \tag{14}$$

The effective-mass form factor is given by

$$\begin{aligned} f_q(\vec{r}) &= \frac{m}{m_q^*(\vec{r})} = \frac{2m}{\hbar^2} \frac{\delta\mathcal{E}(\vec{r})}{\delta\tau_q(\vec{r})} = \\ &= 1 + \frac{2m}{\hbar^2} \left\{ \frac{1}{4} \left[t_1(1+\frac{x_1}{2}) + t_2(1+\frac{x_2}{2}) \right] \rho(\vec{r}) \right. \\ & \left. - \frac{1}{4} \left[t_1(x_1+\frac{1}{2}) - t_2(x_2+\frac{1}{2}) \right] \rho_q(\vec{r}) \right\} \end{aligned} \tag{15}$$

and the spin-orbit potential

$$\vec{W}_q(\vec{r}) = \frac{\delta\mathcal{E}(\vec{r})}{\delta\vec{J}_q(\vec{r})} = \frac{1}{2} W_0 \, \vec{\nabla}(\rho + \rho_q) \; . \tag{16}$$

Out of these different form factors only the single particle potential, eq. (12), depends not only on the local density but also on other quantities, namely the kinetic energy and spin-orbit density. These are used in our approach in their ETF approximations, eqs (2) and (6). It has been shown in [12] that the $4^{\rm th}$ order contributions to the local functions $\tau(r)$ and $\vec{J}(r)$ are essentially negligible, so that the TF and $2^{\rm nd}$ order contributions eqs (3), (4) and (7) determine the densities $\tau_{ETF}[\rho]$ and $\vec{J}_{ETF}[\rho]$ to a very high accuracy, using eq. (13) the single particle potential is given as a function of the local densities and their derivatives.

The above discussed form factors such as the single-particle potential, effective mass, spin-orbit and Coulomb potential, eqs (13)-(16), are easy to use analytic functions of the local densities and their derivatives. These densities can be constructed for any desired nucleus throughout the periodic table in the form given in eq. (8) with the parameters ρ, R, a, γ obtained directly from a density variational calculation, or using the analytic form presented in section 3.1 below. We consider this as the most accurate way to obtain the ingredients of the Skyrme HF equation (11). However, for the reader who is accustomed to work with phenomenological single particle potentials like those of the Woods-Saxon form, the above procedure might still look too cumbersome. This is why we give in the next section a parametrization of these potentials in an analytic way.

3. PARAMETRIZATION OF FORM FACTORS

The aim now is to find easy to use analytical forms which fit as well as possible the self-consistent form factors we just obtained. It is not so surprising that the nuclear central mean field potentials obtained in this way for a given nucleus and a given Skyrme interaction [12] can be well approximated by the familiar Woods-Saxon form

$$V(r) = \frac{V_c}{1+\exp(\frac{r-R}{a^v})} \tag{17}$$

where the parameters V_c, R and a^v are adjusted by a least-square fit procedure such that they reproduce the best possible the nuclear part of the Skyrme central potential (13) (i.e. leaving out the Coulomb potential for protons which is treated separately). Such a fitting procedure is carried out independently for neutrons and protons and separately for each nucleus.

As the Skyrme spin-orbit potential is given as the sum of gradients of nuclear densities one can write

$$\vec{W}(\vec{r}) = \vec{\nabla} V^{so}(\vec{r}) \tag{18}$$

with

$$V^{so}(\vec{r}) = \frac{V_c^{so}}{1+\exp(\frac{r-R^{so}}{a^{so}})} \,. \tag{19}$$

The effective mass is parametrized in the simillar fashion. The formula reads

$$\frac{m^*(\vec{r})}{m} = 1 - \frac{m_c}{1+\exp(\frac{r-R^m}{a^m})} \,. \tag{20}$$

The energy of the Coulomb interaction, eq. 14, is assumed in the form of the of sharp spherical density distribution of Z protons with adjustable radius R_C

$$V^C(r) = \begin{cases} \frac{e^2 Z}{2R^C}\left(3 - \left(\frac{r}{R^C}\right)^2\right) & \text{if } r < R^C \\ \frac{e^2 Z}{r} & \text{if } r \geq R^C \end{cases} \tag{21}$$

The central nuclear field, the spin-orbit and the Coulomb potentials as well as the effective mass are displayed for all three Skyrme forces used here in the sections corresponding to each of the interactions: SIII, SkM* and SLy4. The parameters which define each of these analytical functions for each of the interactions are displayed in Tables 34 and 35.

Using the average mean-field potential obtained as described in Eqs. (18-21) with the parameters taken from Tables 15-18 we have evaluated the single particle levels for some double magic nuclei. The level scheme obtained in this way for ^{208}Pb with the Skyrme SLy4 interaction is compared in Fig. 1 with the one resulting from an exact solution of the HF equation. One observes a relatively good agreement between the corresponding spectra.

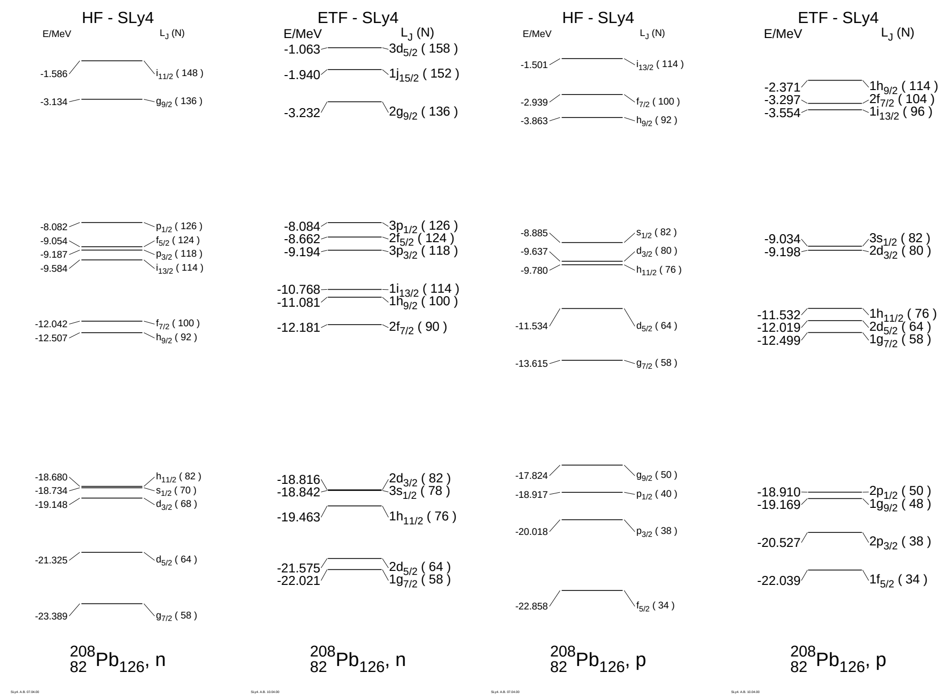

Fig. 1. Exact Skyrme-Hartree-Fock (SLy4) and Extended Thomas-Fermi (ETF) spectra in ^{208}Pb for neutrons (n) and protons (p) in the case of SLy4 Skyrme force. The numbers in parenthesis on the right side of the spectroscopic label $[L_J]$ show the number of particles filling all the levels up to the given one inclusively. All magic gaps at $N = Z = 50$, $N = Z = 82$ and $N = 126$ are reproduced correctly. The neutron and proton binding energies are reproduced as well. (We acknowledge dr. Z. Łojewski for the calculations of the ETF spectra in the corresponding modified Wood-Saxon potential.)

3.1. Z, N PARAMETRIZATION

The form factors obtained separately for magic and superheavy nuclei as described in the previous sections depend on atomic number Z as well as the nucleon number N. In order to have effective and flexible formulae valid for any region of nuclei one has to find the form of this dependence. In the following section we show the way in which we have parametrized the constants entering the form factors.

The quality of the Z, N–parametrization is shown in the corresponding figures and the resulting parameters are displayed in Tables 34 and 35.

Densities

The density

$$\rho(r) = \frac{\rho_c}{\left(1 + \exp\frac{r - R^\rho}{a^\rho}\right)^\gamma}, \tag{22}$$

depends on four constants: ρ_c, R^ρ, a^ρ and γ. Their dependence on Z and N was assumed to be the following

$$\rho_c = \rho_0(1 + \kappa^\rho I), \tag{23}$$

$$R^\rho = r_0^\rho A^{1/3}(1 + \kappa_R^\rho I + b_R^\rho/A), \tag{24}$$

$$a^\rho = a_0^\rho(1 + \kappa_a^\rho I), \tag{25}$$

$$\gamma = \gamma_0(1 + \kappa^\gamma I + b^\gamma/A), \tag{26}$$

where, as usually

$$I = \frac{N - Z}{N + Z}, \quad A = Z + N. \tag{27}$$

Central potential

In a similar fashion the Z and N-dependence of the central potential parameters (eq. 17) was assumed in the form

$$V_c = V_0(1 + \kappa^v I), \tag{28}$$

$$R = r_0^v A^{1/3}(1 + \kappa_R^v I + b/A), \tag{29}$$

$$a^v = a_0^v(1 + \kappa_a^v I). \tag{30}$$

Spin-orbit potential

Spin-orbit potential parameters (eq. 19) are given by

$$V_c^{so} = V_0^{so}(1 + \kappa^{so} I), \tag{31}$$

$$R^{so} = r_0^{so} A^{1/3}(1 + \kappa_R^{so} I + b_R^{so}/A), \tag{32}$$

$$a^{so} = a_0^{so}(1 + \kappa_a^{so} I). \tag{33}$$

Effective mass

The parameters of the effective mass (eq. 20) are

$$m_c = m_0(1 + \kappa^m I)\,, \tag{34}$$

$$R^m = r_0^m A^{1/3}(1 + \kappa_R^m I + b_R^m/A)\,, \tag{35}$$

$$a^m = a_0^m(1 + \kappa_a^m I)\,, \tag{36}$$

$$m = 939\,\mathrm{MeV/c^2}\,. \tag{37}$$

Coulomb potential

The dependence of the Coulomb potential on Z and A (or N) is given in eq. 21 where R_C depends on Z and A

$$R^C = r_0^C A^{1/3}(1 + \kappa_R^C I + b_R^C/A)\,. \tag{38}$$

and

$$e^2 = 1.43996518\ \mathrm{MeV\,fm}\,. \tag{39}$$

3.2. LIQUID DROP MODEL PARAMETERS

The macroscopic part of the energy (corresponding to the liquid drop model (LDM) energy in a Strutinsky procedure) can be extracted for the Skyrme energy functional from infinite and semi-infinite nuclear matter calculations (see [1]). We have, instead, used the parameters found in ref. [16] for each of these interactions

$$E_{\mathrm{LDM}} = a_v(1 - \kappa_v I^2)A + a_s(1 - \kappa_s I^2)A^{2/3} + E_{\mathrm{Coul}}\,, \tag{40}$$

A contribution to the nuclear curvature and compression energy, both proportional to $A^{1/3}$ has been neglected here since such a contribution is known to be small. The values for the parameters (volume a_v, surface a_s, volume asymmetry κ_v and surface asymmetry κ_s) are displayed for the considered Skyrme forces in Table [2] [1, 16, 17].

3.3. HOW TO CALCULATE THE POTENTIALS?

There are two different possibilities of calculating the potentials (central: V, spin-orbit: V^{so} and Coulomb: V^C) as well as the effective mass: m^* for a given Z and N. It can be done in the following ways.

Table 2. Liquid drop model parameters

Force	a_v [MeV]	κ_v	a_s [MeV]	κ_s	r_0 [fm]
SkIII	−15.6342	1.7415	17.7156	1.6064	1.18
SkM*	−15.453	1.748	16.961	2.020	1.14
SLy4	−15.6773	1.8495	17.8839	2.0749	1.14

1. Starting from the set of equations (23-26) and the peculiar density parameters given in Tables 34 and 35 one calculates the corresponding parameters ρ_c, R^ρ, a^ρ and γ^ρ. Then from the formula 22 one calculates the densities as a function of central distance r. In the following step one can apply the formulae for the central potential, eq. 13, the Coulomb potential, eq. 14, spin-orbit, eq. 16 and the effective mass, eq. 16. To do this one uses the Skyrme force parameters displayed in section 1.
2. The second way is easier and consist in direct application of the formulae 17–39 together with calculated peculiar parameters displayed in Tables 34 and 35.

4. DESCRIPTION OF VARIABLES USED IN TABLES AND FIGURES

Variable	Unit	Description
a^m	fm	diffusivity of the effective mass m^*/m (fm)
a_0^m	fm	constant part of the mass diffusivity parameter a_c^m
a^ρ	fm	diffusivity of the density ρ
a_0^ρ	fm	constant part of the density diffusivity a_c^ρ
a^{so}	fm	diffusivity of the SO potential V^{so}
a_0^{so}	fm	constant part of the SO potential diffusivity a^{so}
a^v	fm	diffusivity of the central potential V
a_0^v	fm	constant part of the diffusivity of the central potential a^v
b_R^v		b-parameter of the central potential radius
b_R^{so}		b-parameter of SO potential radius
b^C		b-parameter of the Coulomb radius
b_R^m		b-parameter of the mass radius
b_R^ρ		b-parameter of the density radius

r_0	fm	central potential radius constant
r_0^{so}	fm	SO potential radius constant
r_0^C	fm	Coulomb potential radius constant
r_0^m	fm	effective mass radius constant
r_0^ρ	fm	density radius constant
R	fm	radius of the central potential
R^C	fm	corresponding sharp radius of the Coulomb potential V^C (fm)
R^m	fm	radius of the effective mass
R^{so}	fm	radius of SO potential
R^ρ	fm	radius of the density distribution
κ^v		asymmetry of the central potential depth V_c
κ_a^v		asymmetry of the central potential diffusivity a^v
κ_R^v		asymmetry of the central potential radius R
κ^{so}		asymmetry of the SO potential depth V_c^{so}
κ_a^{so}		asymmetry of the SO potential diffusivity a^v
κ_R^{so}		asymmetry of the SO potential radius R^{so}
κ^m		asymmetry of the effective mass parameter m_c
κ_a^m		asymmetry of the effective mass diffusivity a^m
κ_R^m		asymmetry of the effective mass radius R^m
κ^C		asymmetry of the Coulomb potential radius R^C
κ_c^C		asymmetry of the Coulomb potential modifier c- parameter
κ^ρ		asymmetry of the density parameter ρ_c
κ_a^ρ		asymmetry of the density diffusivity parameter a^ρ
κ_R^ρ		asymmetry of the density radius R^ρ
κ^γ		asymmetry of the density exponent γ
γ	–	density exponent
γ_0	–	constant part of the density exponent γ
m	MeV/c^2	free nucleon mass
m_c	MeV/c^2	*central* value of the effective mass parameter m_c
m_0	MeV/c^2	constant part of the mass parameter m_c
m^*	MeV/c^2	effective nucleon mass
V	MeV	central potential
V_c	MeV	central potential depth parameter
V_0	MeV	constant part of the central potential depth V_c
V^{so}	MeV	SO potential
V_c^{so}	MeV	depth parameter of the SO potential V^{so}
V_0^{so}	MeV	constant part of the SO potential parameter V_c^{so}
V^C	MeV	Coulomb potential

ρ	$1/\mathrm{fm}^3$	nuclear density
ρ_c	$1/\mathrm{fm}^3$	*central* value of the nuclear density
ρ_0	$1/\mathrm{fm}^3$	constant part of the *central* value of the density
W	MeV/fm	derivative of the SO potential V^{so} for neutrons
Z		proton number
N		neutron number
A		mass number
I		asymmetry $(N-Z)/A$
p (•)		*protons*
n (○)		*neutrons*

5. STABLE NUCLEI

5.1. SkIII

Density. SkIII

Table 4. Density parameters. SkIII

		Protons				Neutrons			
Z	N	ρ_c	R^ρ	a^ρ	γ	ρ_c	R^ρ	a^ρ	γ
8	8	0.077217	2.82636	0.46658	1.1883	0.079145	2.82422	0.47134	1.2253
20	20	0.075171	3.96545	0.46539	1.1962	0.077983	3.96301	0.47688	1.2751
20	28	0.066158	4.16605	0.44076	1.2157	0.086980	4.24824	0.50647	1.2332
28	28	0.074066	4.47055	0.46234	1.1834	0.077230	4.46914	0.47815	1.2868
40	40	0.072764	5.06863	0.45736	1.1602	0.076318	5.06949	0.47931	1.2968
40	50	0.066748	5.24080	0.44069	1.1853	0.082687	5.30389	0.49798	1.2754
50	82	0.058328	5.91105	0.42126	1.1724	0.089726	6.03886	0.52332	1.1989
58	82	0.061886	6.08366	0.42605	1.1608	0.085332	6.18474	0.50989	1.2561
82	126	0.058169	6.96056	0.41305	1.1265	0.086825	7.08722	0.51794	1.2356

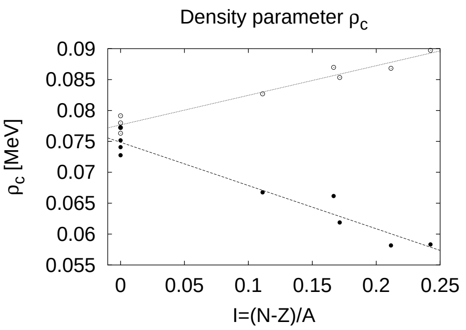

Fig. 2

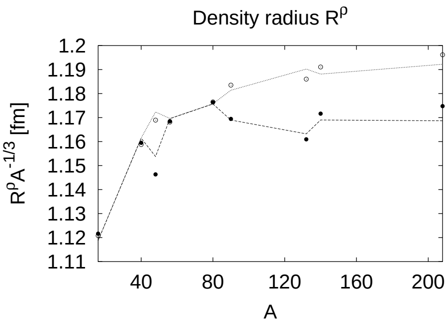

Fig. 3

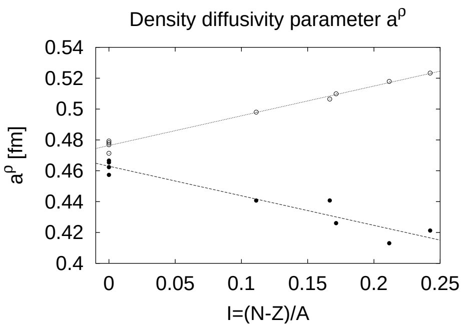

Fig. 4

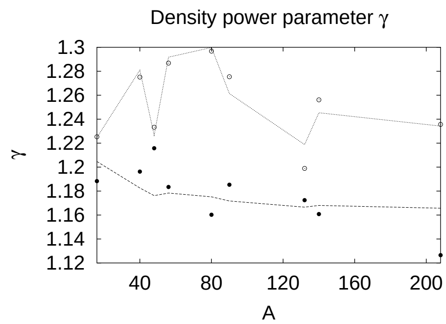

Fig. 5

Potentials. SkIII

Table 5. Central potential parameters. SkIII

		Protons			Neutrons		
Z	N	V_c [MeV]	R [fm]	a^v [fm]	V_c [MeV]	R [fm]	a^v [fm]
8	8	−60.2172	3.0739	0.5570	−59.4804	3.0938	0.5559
20	20	−61.3402	4.1927	0.5505	−60.1487	4.2355	0.5447
20	28	−65.3306	4.5358	0.5495	−56.8125	4.4076	0.5469
28	28	−61.5693	4.6922	0.5495	−60.1972	4.7470	0.5411
40	40	−61.7685	5.2849	0.5496	−60.1908	5.3549	0.5379
40	50	−64.5927	5.5485	0.5481	−57.7043	5.5003	0.5388
50	82	−68.6023	6.3540	0.5544	−54.7975	6.1720	0.5443
58	82	−66.6073	6.4454	0.5508	−56.2090	6.3481	0.5384
82	126	−68.1231	7.3598	0.5555	−55.1020	7.2360	0.5387

Table 6. Spin-orbit potential parameters. SkIII

		Protons			Neutrons		
Z	N	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]
8	8	−14.1586	2.7137	0.4443	−14.0415	2.7205	0.4440
20	20	−13.8989	3.8352	0.4441	−13.7276	3.8490	0.4430
20	28	−14.4229	4.0999	0.4591	−13.1679	4.0806	0.4390
28	28	−13.7417	4.3394	0.4437	−13.5486	4.3570	0.4421
40	40	−13.5528	4.9398	0.4432	−13.3353	4.9622	0.4407
40	50	−13.9493	5.1539	0.4507	−12.9876	5.1514	0.4356
50	82	−14.2620	5.8937	0.4709	−12.3646	5.8621	0.4395
58	82	−13.9630	6.0312	0.4572	−12.5475	6.0187	0.4340
82	126	−13.9068	6.9369	0.4623	−12.1746	6.9190	0.4329

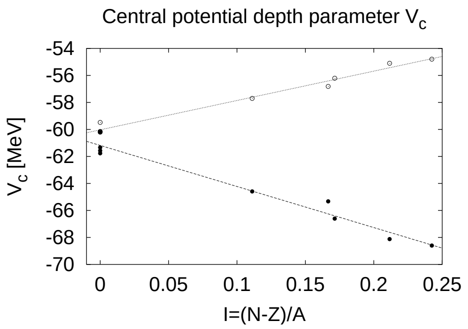

Fig. 6

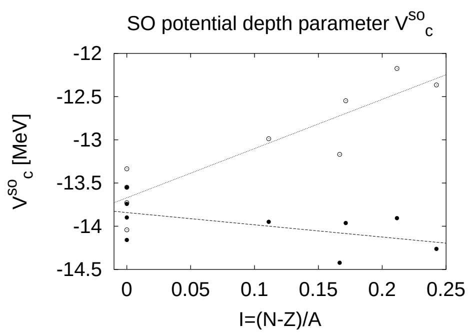

Fig. 7

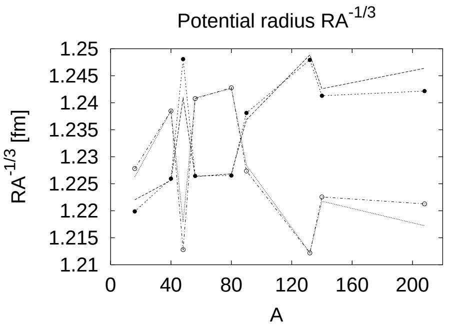

Fig. 8

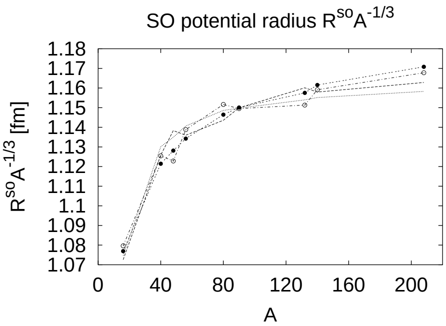

Fig. 9

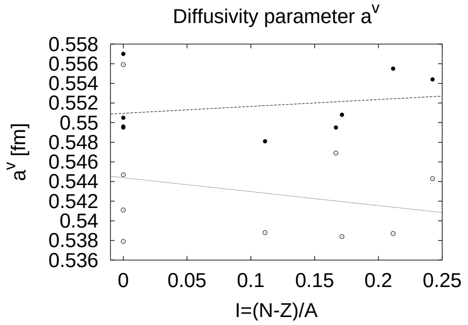

Fig. 10

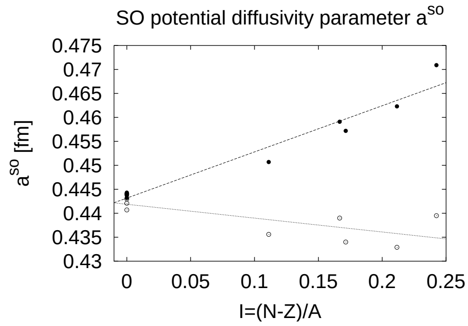

Fig. 11

Effective mass. SkIII

Table 7. Effective mass parameters. SkIII

		Protons			Neutrons		
Z	N	m_c	R^m [fm]	a^m [fm]	m_c	R^m [fm]	a^m [fm]
8	8	0.2512	2.8232	0.4341	0.2482	2.8369	0.4347
20	20	0.2485	3.9338	0.4349	0.2439	3.9629	0.4345
20	28	0.2632	4.2340	0.4601	0.2285	4.1643	0.4210
28	28	0.2467	4.4337	0.4351	0.2415	4.4712	0.4340
40	40	0.2444	5.0292	0.4354	0.2385	5.0778	0.4330
40	50	0.2551	5.2670	0.4495	0.2282	5.2435	0.4212
50	82	0.2661	6.0423	0.4791	0.2127	5.9328	0.4179
58	82	0.2587	6.1557	0.4608	0.2188	6.1008	0.4163
82	126	0.2604	7.0689	0.4696	0.2112	6.9959	0.4131

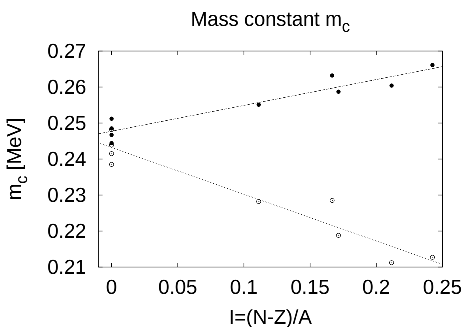

Fig. 12

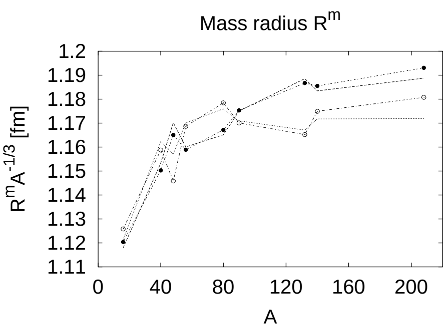

Fig. 13

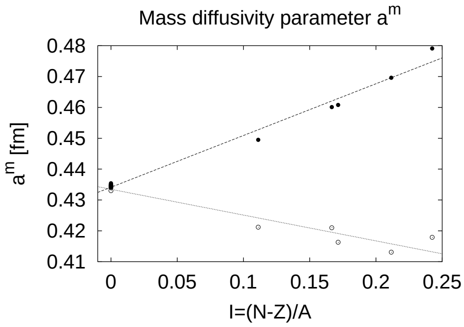

Fig. 14

Coulomb potential. SkIII

Table 8. Coulomb potential parameters. SkIII

Z	N	R^C
8	8	3.1976
20	20	4.2197
20	28	4.3670
28	28	4.7034
40	40	5.2927
40	50	5.4276
50	82	6.0839
58	82	6.2675
82	126	7.1643

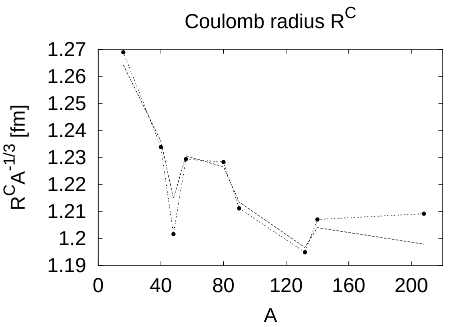

Fig. 15

5.2. SkM*

*Density. SkM**

Table 9. Density parameters. SkM*

		Protons				Neutrons			
Z	N	ρ_c	R^ρ	a^ρ	γ	ρ_c	R^ρ	a^ρ	γ
8	8	0.081275	2.87789	0.57594	1.4180	0.083456	2.87486	0.57840	1.4551
20	20	0.081808	4.01396	0.59517	1.4957	0.085292	4.00923	0.60310	1.5806
20	28	0.073143	4.18683	0.56707	1.5202	0.093539	4.32896	0.64292	1.5485
28	28	0.080579	4.51782	0.59496	1.4913	0.084576	4.51404	0.60705	1.6055
40	40	0.078779	5.11393	0.59075	1.4673	0.083334	5.11298	0.60922	1.6225
40	50	0.072938	5.26479	0.56968	1.4972	0.089342	5.36954	0.63298	1.6043
50	82	0.063485	5.91667	0.54638	1.4720	0.094625	6.14807	0.67023	1.5198
58	82	0.067071	6.09482	0.55055	1.4588	0.090876	6.26815	0.64807	1.5816
82	126	0.062207	6.96787	0.53174	1.4020	0.091091	7.18972	0.65688	1.5497

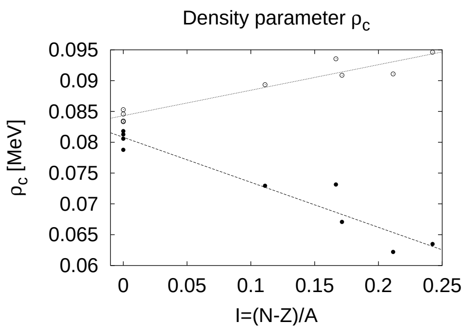

Fig. 16

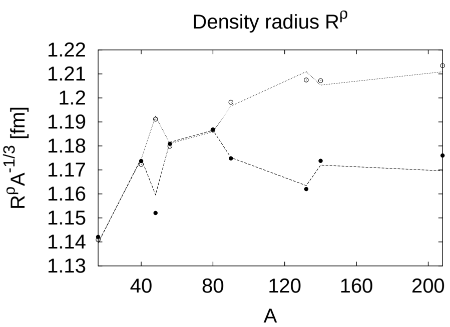

Fig. 17

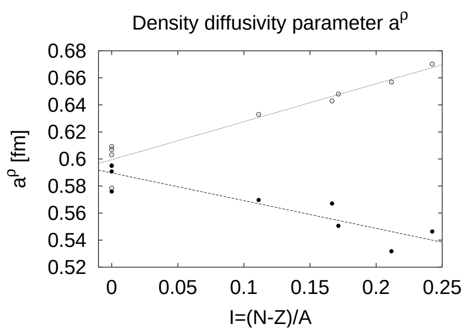

Fig. 18

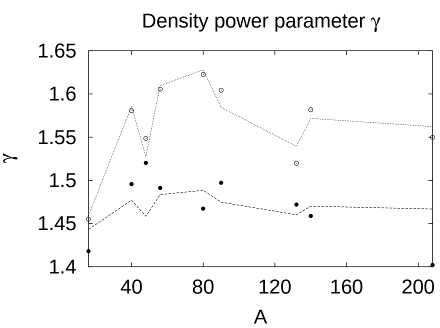

Fig. 19

*Potentials. SkM**

Table 10. Central potential parameters. SkM*

		Protons			Neutrons		
Z	N	V_c [MeV]	R [fm]	a^v [fm]	V_c [MeV]	R [fm]	a^v [fm]
8	8	−61.2802	3.0063	0.6917	−60.3537	3.0310	0.6911
20	20	−63.5751	4.0828	0.6962	−61.9357	4.1387	0.6895
20	28	−67.3578	4.4258	0.7017	−58.3585	4.3207	0.6926
28	28	−63.7315	4.5784	0.6948	−61.8244	4.6499	0.6849
40	40	−63.6993	5.1707	0.6933	−61.4988	5.2622	0.6791
40	50	−66.4625	5.4324	0.6944	−58.8594	5.4114	0.6777
50	82	−69.6928	6.2623	0.7100	−55.0303	6.1177	0.6873
58	82	−67.9362	6.3403	0.6980	−56.6778	6.2781	0.6744
82	126	−68.8176	7.2747	0.7026	−54.8760	7.1928	0.6720

Table 11. Spin-orbit potential parameters. SkM*

		Protons			Neutrons		
Z	N	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]
8	8	−16.1754	2.6203	0.5220	−16.0325	2.6275	0.5223
20	20	−16.4580	3.6936	0.5337	−16.2301	3.7094	0.5337
20	28	−16.9568	3.9626	0.5527	−15.6238	3.9324	0.5309
28	28	−16.2847	4.1899	0.5350	−16.0232	4.2104	0.5345
40	40	−16.0049	4.7863	0.5351	−15.7066	4.8131	0.5338
40	50	−16.4010	5.0041	0.5436	−15.3298	4.9963	0.5278
50	82	−16.4537	5.7646	0.5704	−14.4117	5.7144	0.5349
58	82	−16.2087	5.8909	0.5512	−14.6520	5.8681	0.5256
82	126	−15.9094	6.8154	0.5564	−14.0173	6.7839	0.5232

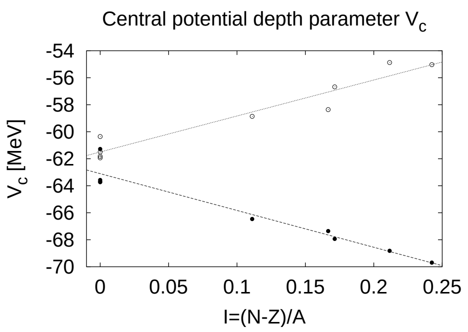

Fig. 20

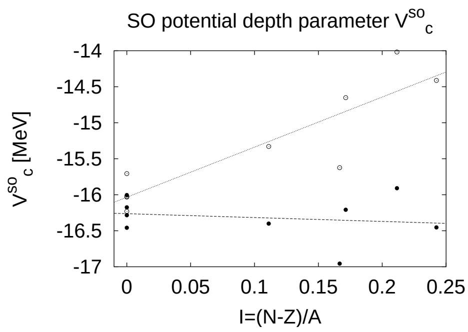

Fig. 21

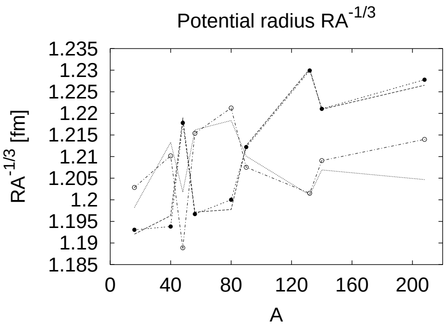

Fig. 22

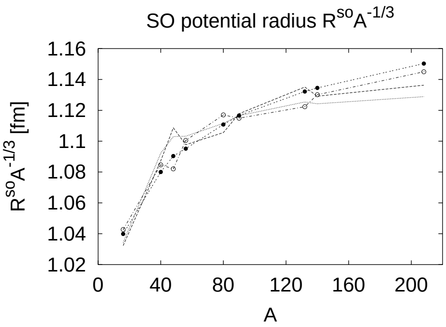

Fig. 23

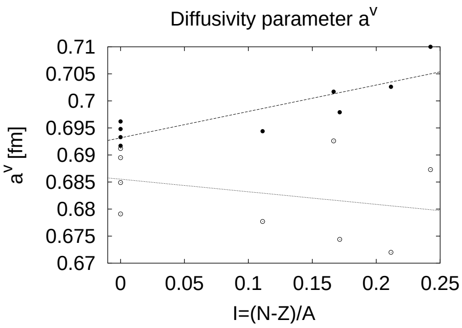

Fig. 24

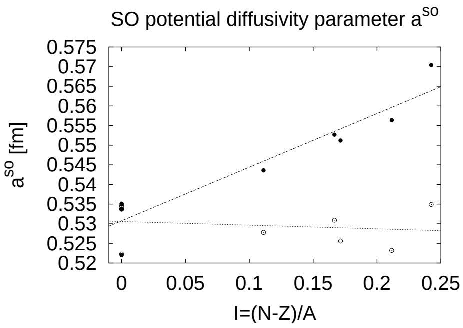

Fig. 25

*Effective mass. SkM**

Table 12. Effective mass parameters. SkM*

		Protons			Neutrons		
Z	N	m_c	R^m [fm]	a^m [fm]	m_c	R^m [fm]	a^m [fm]
8	8	0.2158	2.7140	0.4991	0.2117	2.7340	0.5016
20	20	0.2203	3.7706	0.5123	0.2135	3.8161	0.5145
20	28	0.2363	4.1021	0.5485	0.1959	3.9748	0.4904
28	28	0.2191	4.2582	0.5151	0.2113	4.3186	0.5164
40	40	0.2168	4.8443	0.5170	0.2078	4.9247	0.5167
40	50	0.2287	5.1071	0.5380	0.1957	5.0595	0.4967
50	82	0.2390	5.9244	0.5787	0.1753	5.7204	0.4814
58	82	0.2318	6.0118	0.5540	0.1832	5.9073	0.4856
82	126	0.2323	6.9468	0.5657	0.1725	6.8088	0.4758

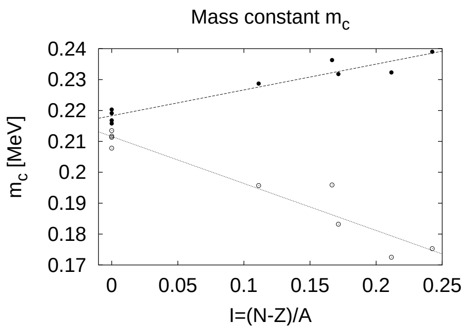

Fig. 26

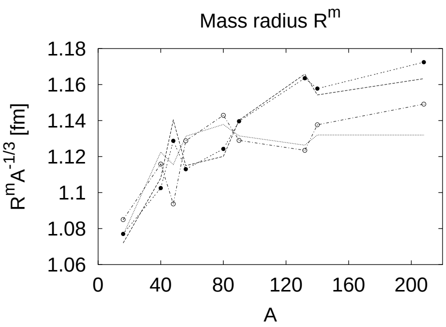

Fig. 27

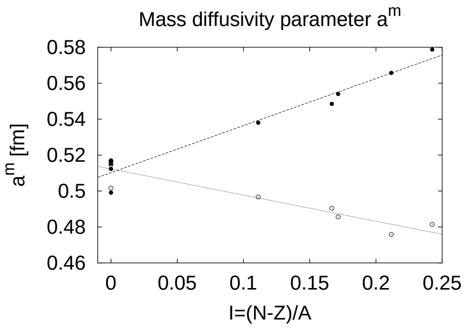

Fig. 28

*Coulomb potential. SkM**

Table 13. Coulomb potential parameters. SkM*

Z	N	R^C
8	8	3.2302
20	20	4.1869
20	28	4.3021
28	28	4.6517
40	40	5.2251
40	50	5.3346
50	82	5.9696
58	82	6.1551
82	126	7.0492

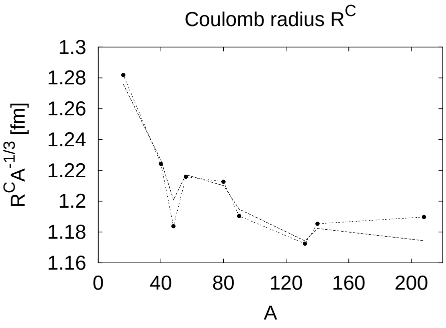

Fig. 29

5.3. SLy4

Density. SLy4

Table 14. Density parameters. SLy4

		Protons				Neutrons			
Z	N	ρ_c	R^ρ	a^ρ	γ	ρ_c	R^ρ	a^ρ	γ
8	8	0.079094	2.89510	0.56819	1.3969	0.081288	2.89028	0.57163	1.4334
20	20	0.080299	4.03323	0.58984	1.4849	0.083693	4.02794	0.59929	1.5688
20	28	0.071614	4.22093	0.56703	1.5275	0.092124	4.33167	0.63320	1.5175
28	28	0.079382	4.53825	0.59123	1.4872	0.083243	4.53396	0.60477	1.5990
40	40	0.077891	5.13438	0.58835	1.4690	0.082270	5.13399	0.60861	1.6216
40	50	0.071982	5.29550	0.57090	1.5101	0.088402	5.37929	0.62861	1.5902
50	82	0.062639	5.96024	0.55164	1.5014	0.093993	6.13827	0.65922	1.4865
58	82	0.066360	6.13184	0.55505	1.4848	0.090264	6.27002	0.64244	1.5637
82	126	0.061710	7.00705	0.53826	1.4361	0.090762	7.18385	0.65034	1.5291
82	126	0.061710	7.00705	0.53826	1.4361	0.090762	7.18385	0.65034	1.5291

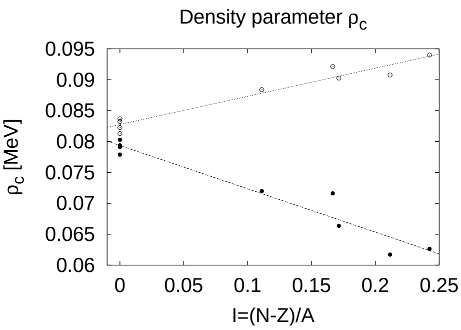

Fig. 30

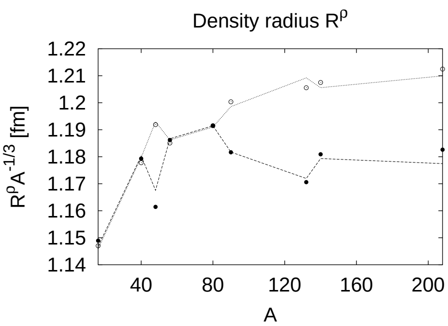

Fig. 31

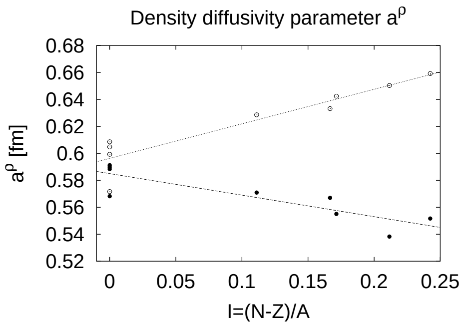

Fig. 32

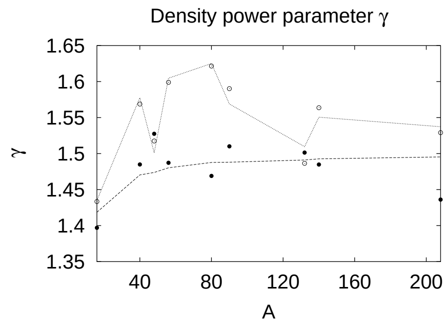

Fig. 33

Potentials. SLy4

Table 15. Central potential parameters. SLy4.

		Protons			Neutrons		
Z	N	V_c [MeV]	R [fm]	a^v [fm]	V_c [MeV]	R [fm]	a^v [fm]
8	8	−66.8758	2.9284	0.7121	−66.5314	2.9376	0.7179
20	20	−69.5833	4.0099	0.7147	−68.7986	4.0412	0.7203
20	28	−71.8495	4.3948	0.7334	−67.0354	4.1903	0.7032
28	28	−69.6615	4.5099	0.7112	−68.7127	4.5526	0.7170
40	40	−69.4801	5.1072	0.7070	−68.3445	5.1653	0.7126
40	50	−71.2081	5.3955	0.7185	−66.9920	5.2878	0.6984
50	82	−73.1742	6.2679	0.7382	−64.6894	5.9654	0.6893
58	82	−71.9633	6.3251	0.7231	−65.4713	6.1388	0.6886
82	126	−72.3719	7.2738	0.7268	−64.0503	7.0405	0.6820

Table 16. Spin-orbit potential parameters. SLy4.

		Protons			Neutrons		
Z	N	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]
8	8	−14.9026	2.6495	0.5175	−14.7666	2.6573	0.5176
20	20	−15.2812	3.7200	0.5308	−15.0710	3.7360	0.5303
20	28	−15.7754	3.9883	0.5483	−14.5080	3.9609	0.5284
28	28	−15.1695	4.2141	0.5330	−14.9303	4.2346	0.5320
40	40	−14.9570	4.8079	0.5340	−14.6856	4.8344	0.5321
40	50	−15.3435	5.0246	0.5418	−14.3292	5.0187	0.5268
50	82	−15.4391	5.7829	0.5661	−13.4965	5.7382	0.5337
58	82	−15.2179	5.9077	0.5493	−13.7398	5.8883	0.5256
82	126	−14.9834	6.8276	0.5543	−13.1846	6.8007	0.5236

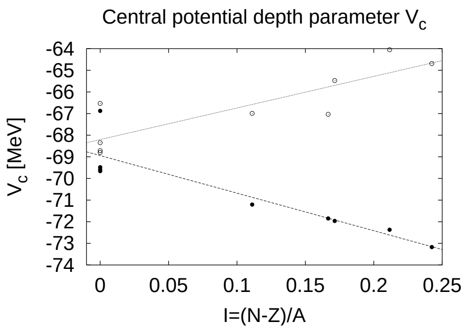

Fig. 34

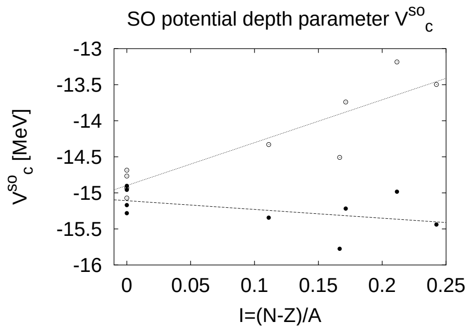

Fig. 35

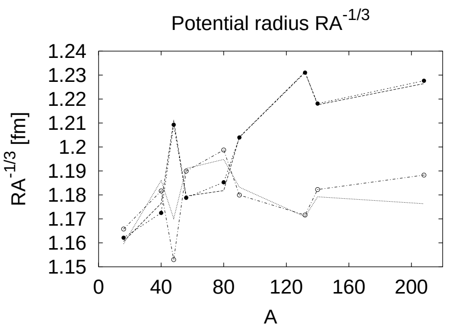

Fig. 36

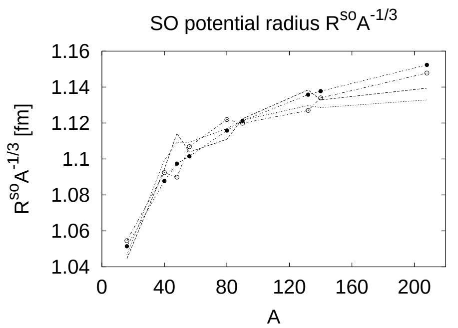

Fig. 37

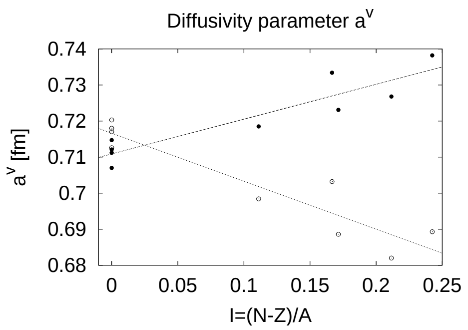

Fig. 38

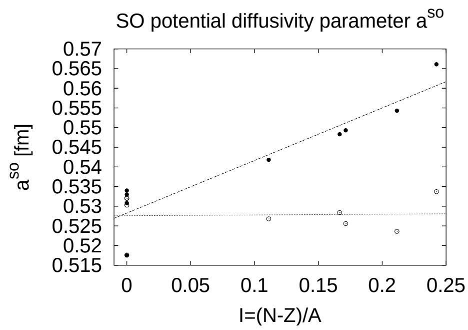

Fig. 39

Effective mass. SLy4

Table 17. Effective mass parameters. SLy4.

		Protons			Neutrons		
Z	N	m_c	R^m [fm]	a^m [fm]	m_c	R^m [fm]	a^m [fm]
8	8	0.3023	2.8193	0.4943	0.3047	2.8103	0.4935
20	20	0.3072	3.8943	0.5072	0.3109	3.8749	0.5066
20	28	0.2973	4.1041	0.5049	0.3203	4.1652	0.5278
28	28	0.3055	4.3901	0.5100	0.3097	4.3647	0.5097
40	40	0.3023	4.9869	0.5115	0.3071	4.9534	0.5120
40	50	0.2957	5.1581	0.5052	0.3142	5.1835	0.5217
50	82	0.2819	5.8694	0.5149	0.3181	5.9662	0.5521
58	82	0.2864	6.0195	0.5058	0.3139	6.0717	0.5326
82	126	0.2776	6.9269	0.5060	0.3117	6.9949	0.5410

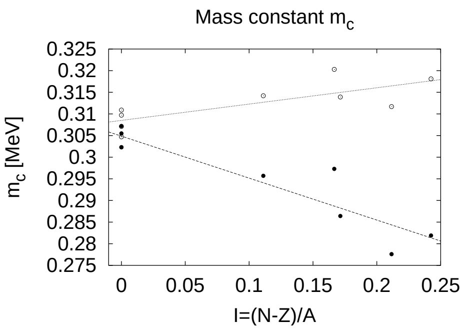

Fig. 40

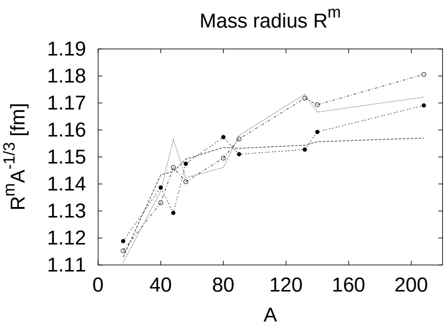

Fig. 41

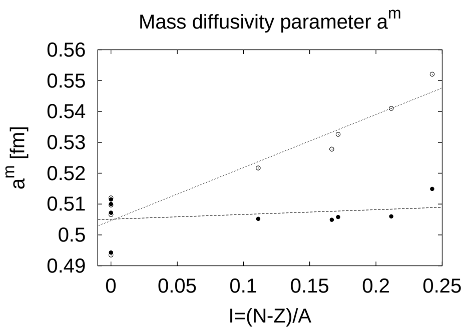

Fig. 42

Coulomb potential. SLy4

Table 18. Coulomb potential parameters. SLy4.

Z	N	R_0
8	8	3.2492
20	20	4.2067
20	28	4.3288
28	28	4.6703
40	40	5.2417
40	50	5.3565
50	82	5.9964
58	82	6.1771
82	126	7.0690

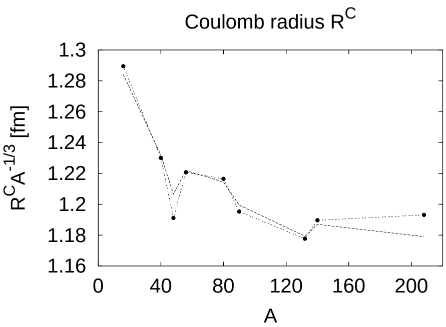

Fig. 43

6. SUPERHEAVY NUCLEI

6.1. SkIII

Density. SkIII

Table 19. Density parameters. SkIII

		Protons				Neutrons			
Z	N	ρ_c	R^ρ	a^ρ	γ	ρ_c	R^ρ	a^ρ	γ
110	144	0.061283	7.52013	0.41243	1.0797	0.081234	7.61131	0.50316	1.2855
110	154	0.059470	7.60083	0.40866	1.0849	0.083244	7.71061	0.50926	1.2701
110	164	0.057756	7.67865	0.40539	1.0880	0.085095	7.80429	0.51509	1.2515
118	154	0.061039	7.70371	0.41002	1.0684	0.080889	7.79518	0.50282	1.2869
118	164	0.059351	7.78126	0.40652	1.0739	0.082785	7.89028	0.50852	1.2729
118	174	0.057747	7.85618	0.40343	1.0772	0.084539	7.98050	0.51399	1.2561
126	164	0.060800	7.87969	0.40763	1.0573	0.080561	7.97148	0.50249	1.2881
126	174	0.059221	7.95449	0.40441	1.0631	0.082356	8.06280	0.50786	1.2754
126	184	0.057715	8.02670	0.40147	1.0666	0.084023	8.14978	0.51300	1.2601

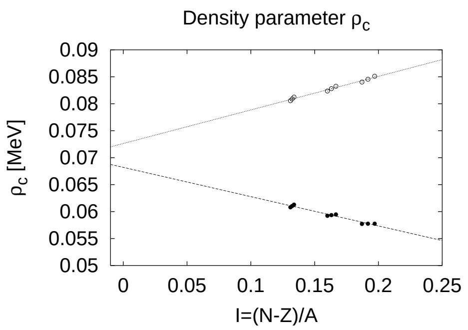

Fig. 44

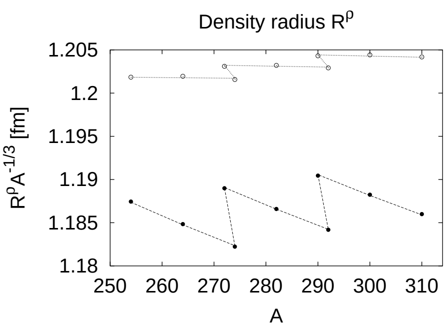

Fig. 45

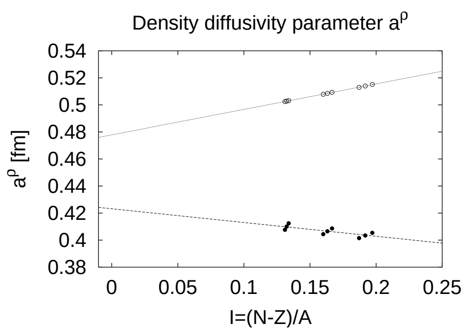

Fig. 46

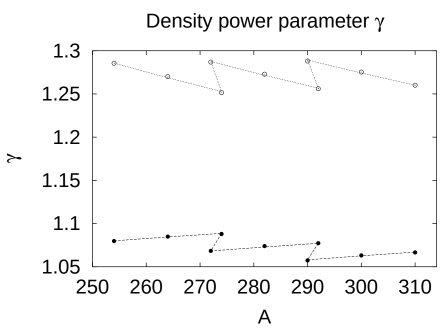

Fig. 47

Potentials. SkIII

Table 20. Central potential parameters. SkIII

		Protons			Neutrons		
Z	N	V_c [MeV]	R [fm]	a^v [fm]	V_c [MeV]	R [fm]	a^v [fm]
110	144	−65.9376	7.8307	0.5556	−56.6453	7.8147	0.5322
110	154	−66.9374	7.9444	0.5564	−55.8872	7.8910	0.5338
110	164	−67.8882	8.0548	0.5578	−55.1965	7.9665	0.5356
118	154	−65.9264	8.0111	0.5564	−56.6280	8.0023	0.5316
118	164	−66.8673	8.1194	0.5571	−55.9137	8.0753	0.5331
118	174	−67.7648	8.2247	0.5583	−55.2588	8.1477	0.5348
126	164	−65.9153	8.1842	0.5572	−56.6080	8.1822	0.5311
126	174	−66.8047	8.2875	0.5578	−55.9325	8.2522	0.5324
126	184	−67.6547	8.3883	0.5589	−55.3100	8.3217	0.5340

Table 21. Spin-orbit potential parameters. SkIII

		Protons			Neutrons		
Z	N	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]
110	144	−13.4325	7.4653	0.4486	−12.2247	7.4733	0.4268
110	154	−13.5595	7.5617	0.4529	−12.1211	7.5610	0.4273
110	164	−13.6729	7.6552	0.4577	−12.0189	7.6463	0.4286
118	154	−13.3754	7.6505	0.4480	−12.1730	7.6604	0.4259
118	164	−13.4966	7.7426	0.4519	−12.0782	7.7443	0.4263
118	174	−13.6053	7.8321	0.4563	−11.9840	7.8260	0.4273
126	164	−13.3205	7.8282	0.4473	−12.1229	7.8400	0.4251
126	174	−13.4365	7.9164	0.4509	−12.0357	7.9203	0.4253
126	184	−13.5411	8.0023	0.4550	−11.9486	7.9989	0.4262

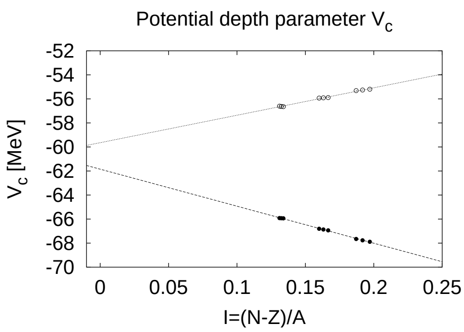

Fig. 48

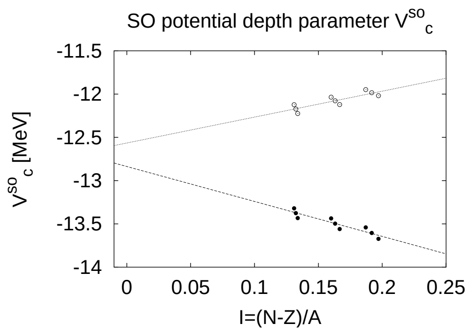

Fig. 49

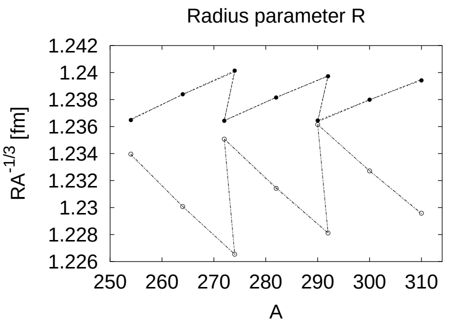

Fig. 50

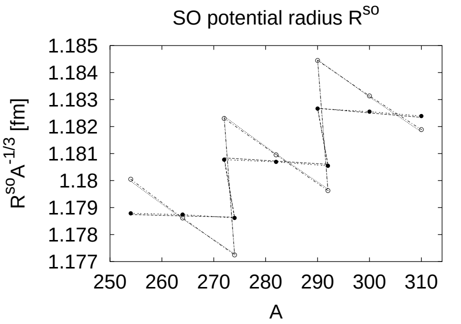

Fig. 51

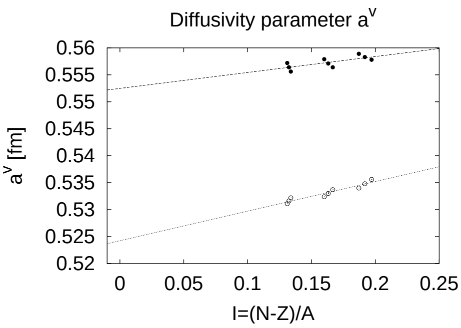

Fig. 52

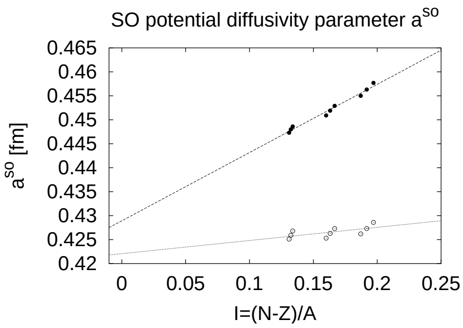

Fig. 53

Efective mass. SkIII

Table 22. Effective mass parameters. SkIII

		Protons			Neutrons		
Z	N	m_c	R^m [fm]	a^m [fm]	m_c	R^m [fm]	a^m [fm]
110	144	0.2503	7.5691	0.4521	0.2159	7.5670	0.4119
110	154	0.2537	7.6749	0.4584	0.2127	7.6481	0.4104
110	164	0.2568	7.7777	0.4652	0.2096	7.7277	0.4098
118	154	0.2496	7.7526	0.4516	0.2152	7.7552	0.4112
118	164	0.2528	7.8534	0.4575	0.2122	7.8327	0.4097
118	174	0.2557	7.9516	0.4638	0.2093	7.9091	0.4090
126	164	0.2488	7.9286	0.4513	0.2146	7.9359	0.4105
126	174	0.2519	8.0249	0.4568	0.2118	8.0102	0.4090
126	184	0.2547	8.1191	0.4626	0.2091	8.0835	0.4082

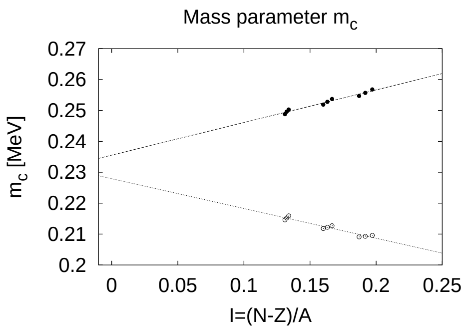

Fig. 54

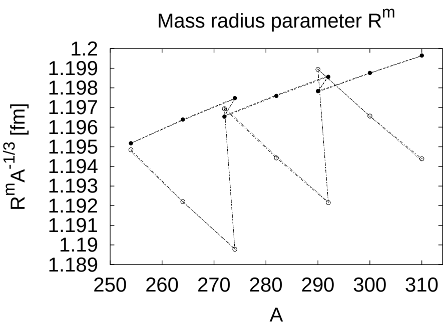

Fig. 55

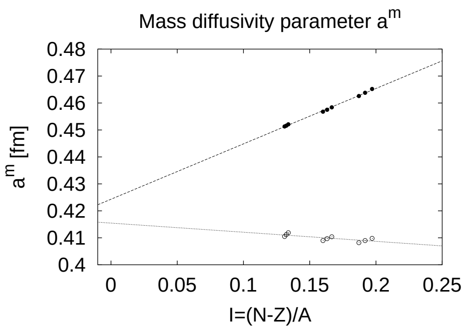

Fig. 56

Coulomb potential. SkIII

Table 23. Coulomb potential parameters. SkIII

Z	N	R^C
110	144	7.7655
110	154	7.8414
110	164	7.9166
118	154	7.9594
118	164	8.0323
118	174	8.1045
126	164	8.1457
126	174	8.2158
126	184	8.2855

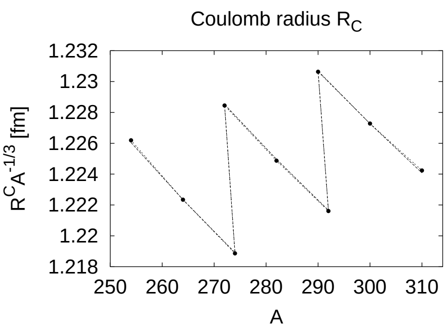

Fig. 57

6.2. SkM*

*Density. SkM**

Table 24. Density parameters. SkM*

		Protons				Neutrons			
Z	N	ρ_c	R^ρ	a^ρ	γ	ρ_c	R^ρ	a^ρ	γ
110	144	0.064712	7.53732	0.52715	1.3346	0.085457	7.69128	0.63192	1.5959
110	154	0.062816	7.61432	0.52221	1.3388	0.087194	7.80189	0.64020	1.5784
110	164	0.060975	7.69017	0.51805	1.3400	0.088708	7.90800	0.64853	1.5569
118	154	0.064233	7.72242	0.52307	1.3165	0.084890	7.87691	0.63042	1.5943
118	164	0.062475	7.79652	0.51844	1.3211	0.086540	7.98261	0.63810	1.5786
118	174	0.060762	7.86959	0.51447	1.3229	0.087991	8.08444	0.64581	1.5592
126	164	0.063767	7.90014	0.51901	1.2986	0.084345	8.05526	0.62898	1.5925
126	174	0.062131	7.97165	0.51468	1.3037	0.085916	8.15648	0.63611	1.5781
126	184	0.060532	8.04214	0.51089	1.3059	0.087310	8.25440	0.64331	1.5606

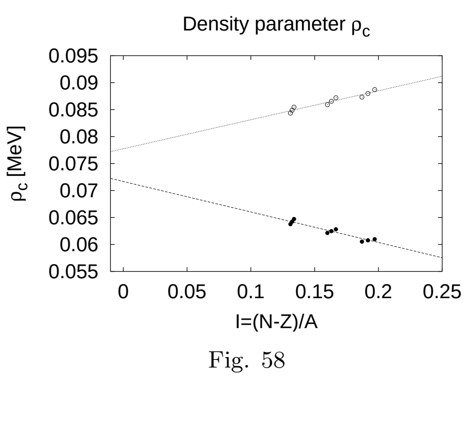

Fig. 58

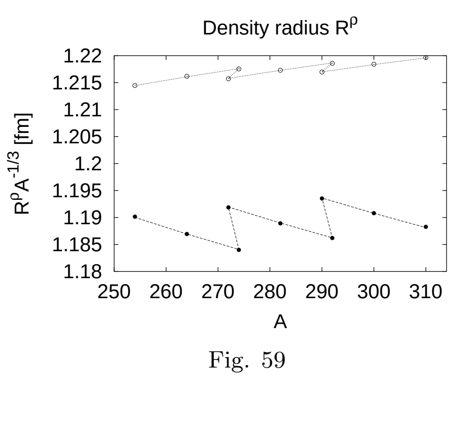

Fig. 59

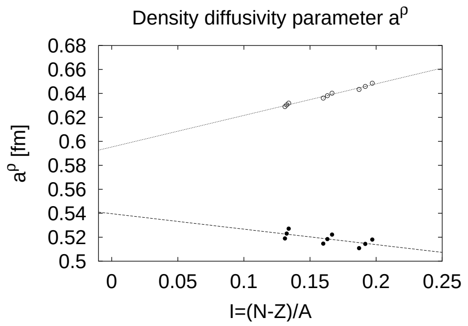

Fig. 60

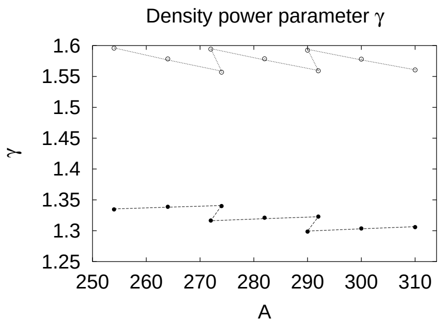

Fig. 61

*Potentials. SkM**

Table 25. Central potential parameters. SkM*

		Protons			Neutrons		
Z	N	V_c [MeV]	R [fm]	a^v [fm]	V_c [MeV]	R [fm]	a^v [fm]
110	144	−66.5184	7.7434	0.6931	−56.3438	7.7706	0.6570
110	154	−67.4074	7.8627	0.6959	−55.4582	7.8537	0.6594
110	164	−68.2145	7.9809	0.6996	−54.6150	7.9385	0.6628
118	154	−66.3893	7.9283	0.6929	−56.2144	7.9637	0.6551
118	164	−67.2307	8.0416	0.6954	−55.3859	8.0430	0.6573
118	174	−67.9989	8.1540	0.6988	−54.5939	8.1238	0.6603
126	164	−66.2629	8.1059	0.6928	−56.0857	8.1494	0.6534
126	174	−67.0617	8.2139	0.6950	−55.3081	8.2252	0.6553
126	184	−67.7950	8.3212	0.6981	−54.5620	8.3024	0.6580

Table 26. SkM*. Spin-orbit potential parameters

		Protons			Neutrons		
Z	N	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]
110	144	−15.3446	7.3428	0.5362	−13.9870	7.3478	0.5123
110	154	−15.4428	7.4451	0.5415	−13.8470	7.4376	0.5131
110	164	−15.5154	7.5461	0.5478	−13.6988	7.5267	0.5150
118	154	−15.2383	7.5325	0.5347	−13.8860	7.5401	0.5105
118	164	−15.3344	7.6301	0.5395	−13.7588	7.6259	0.5111
118	174	−15.4074	7.7266	0.5452	−13.6237	7.7112	0.5127
126	164	−15.1356	7.7149	0.5333	−13.7880	7.7251	0.5088
126	174	−15.2296	7.8083	0.5377	−13.6720	7.8072	0.5092
126	184	−15.3029	7.9006	0.5429	−13.5486	7.8890	0.5105

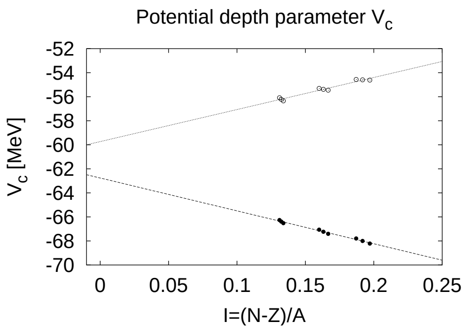

Fig. 62

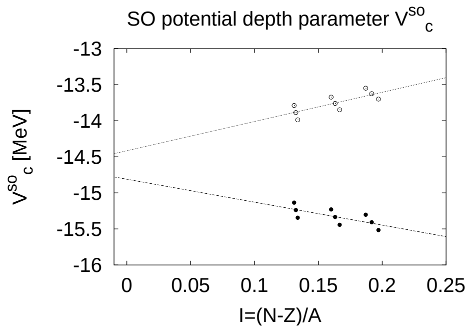

Fig. 63

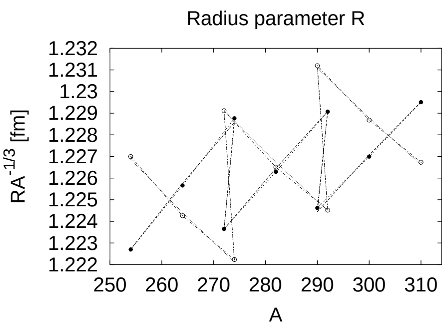

Fig. 64

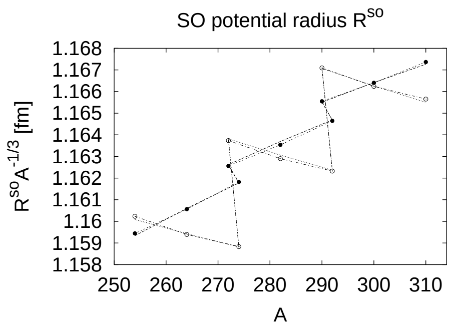

Fig. 65

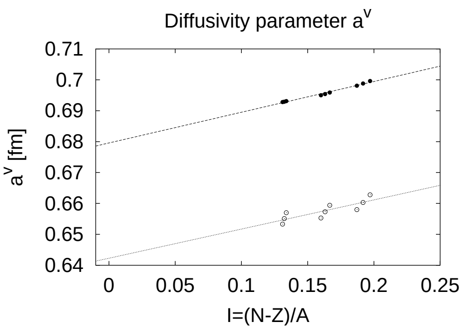

Fig. 66

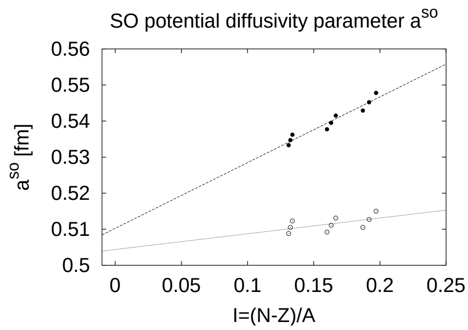

Fig. 67

*Effective mass. SkM**

Table 27. Effective mass parameters. SkM*

		Protons			Neutrons		
Z	N	m_c	R^m [fm]	a^m [fm]	m_c	R^m [fm]	a^m [fm]
110	144	0.2213	7.4248	0.5412	0.1781	7.4171	0.4783
110	154	0.2248	7.5436	0.5499	0.1739	7.4909	0.4737
110	164	0.2278	7.6602	0.5592	0.1698	7.5651	0.4702
118	154	0.2202	7.6110	0.5405	0.1771	7.6123	0.4767
118	164	0.2235	7.7240	0.5486	0.1731	7.6830	0.4724
118	174	0.2264	7.8353	0.5573	0.1693	7.7541	0.4689
126	164	0.2191	7.7901	0.5399	0.1760	7.8001	0.4752
126	174	0.2223	7.8981	0.5475	0.1724	7.8679	0.4710
126	184	0.2250	8.0045	0.5557	0.1688	7.9363	0.4677

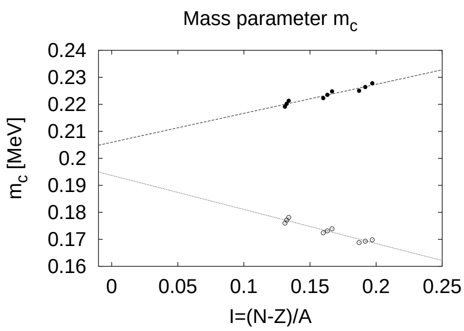

Fig. 68

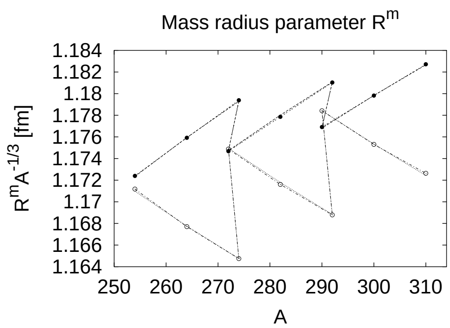

Fig. 69

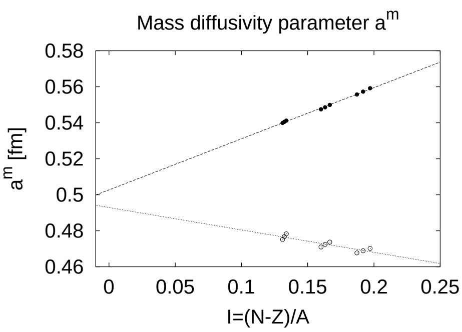

Fig. 70

*Coulomb potential. SkM**

Table 28. Coulomb potential parameters. SkM*

Z	N	R^C
110	144	7.6643
110	154	7.7370
110	164	7.8111
118	154	7.8621
118	164	7.9319
118	174	8.0031
126	164	8.0528
126	174	8.1200
126	184	8.1884

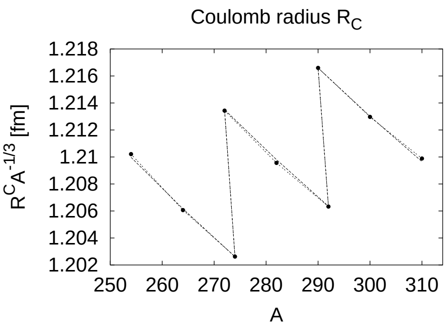

Fig. 71

6.3. SLy4

Density. SLy4

Table 29. Density parameters. SLy4

		Protons				Neutrons			
Z	N	ρ_c	R^ρ	a^ρ	γ	ρ_c	R^ρ	a^ρ	γ
110	144	0.064460	7.56574	0.53253	1.3643	0.085220	7.69499	0.63070	1.5909
110	154	0.062537	7.64596	0.52849	1.3716	0.086999	7.80099	0.63747	1.5687
110	164	0.060677	7.72445	0.52499	1.3752	0.088555	7.90223	0.64414	1.5422
118	154	0.064029	7.74965	0.52859	1.3466	0.084697	7.88011	0.62953	1.5902
118	164	0.062245	7.82665	0.52477	1.3540	0.086386	7.98147	0.63581	1.5700
118	174	0.060513	7.90226	0.52146	1.3582	0.087878	8.07877	0.64203	1.5460
126	164	0.063609	7.92612	0.52463	1.3292	0.084193	8.05787	0.62837	1.5891
126	174	0.061948	8.00031	0.52106	1.3367	0.085802	8.15503	0.63423	1.5707
126	184	0.060330	8.07347	0.51801	1.3417	0.087233	8.24874	0.64006	1.5490

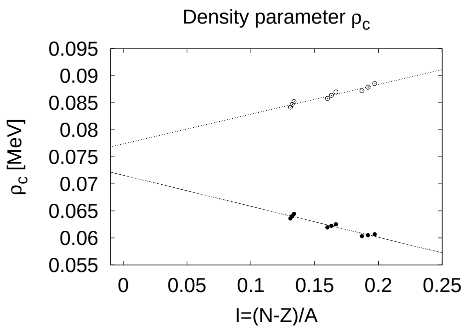

Fig. 72

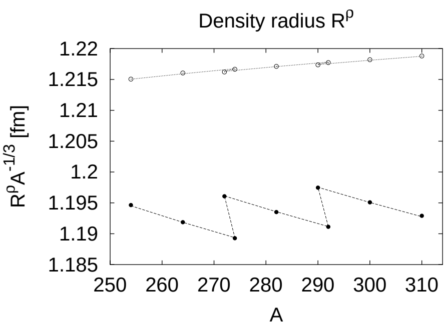

Fig. 73

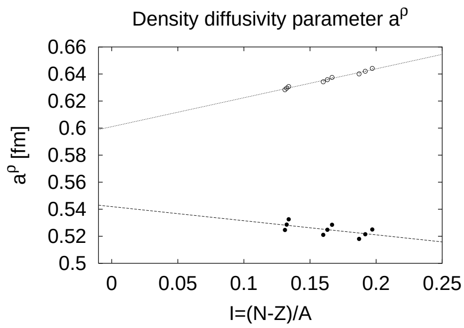

Fig. 74

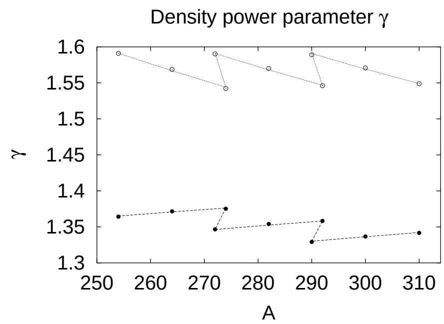

Fig. 75

Potentials. SLy4

Table 30. Central potential parameters. SLy4

		Protons			Neutrons		
Z	N	V_c [MeV]	R [fm]	a^v [fm]	V_c [MeV]	R [fm]	a^v [fm]
110	144	−70.5724	7.7218	0.7104	−64.4142	7.6331	0.6811
110	154	−71.2023	7.8496	0.7157	−63.9371	7.7078	0.6788
110	164	−71.7770	7.9762	0.7212	−63.4719	7.7850	0.6774
118	154	−70.4220	7.9063	0.7095	−64.2228	7.8256	0.6800
118	164	−71.0242	8.0275	0.7144	−63.7782	7.8969	0.6778
118	174	−71.5766	8.1476	0.7195	−63.3436	7.9703	0.6764
126	164	−70.2747	8.0836	0.7086	−64.0348	8.0107	0.6790
126	174	−70.8523	8.1989	0.7132	−63.6194	8.0787	0.6768
126	184	−71.3838	8.3133	0.7181	−63.2119	8.1489	0.6754

Table 31. Spin-orbit potential parameters. SLy4

		Protons			Neutrons		
Z	N	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]	V^{so} [MeV]	R^{so} [fm]	a^{so} [fm]
110	144	−14.4752	7.3512	0.5365	−13.1903	7.3582	0.5135
110	154	−14.5719	7.4531	0.5413	−13.0579	7.4486	0.5143
110	164	−14.6449	7.5535	0.5469	−12.9186	7.5383	0.5161
118	154	−14.3830	7.5397	0.5352	−13.1035	7.5492	0.5119
118	164	−14.4775	7.6368	0.5395	−12.9830	7.6355	0.5125
118	174	−14.5507	7.7328	0.5446	−12.8560	7.7212	0.5139
126	164	−14.2938	7.7209	0.5339	−13.0191	7.7328	0.5102
126	174	−14.3862	7.8137	0.5379	−12.9092	7.8154	0.5107
126	184	−14.4592	7.9056	0.5425	−12.7930	7.8975	0.5119

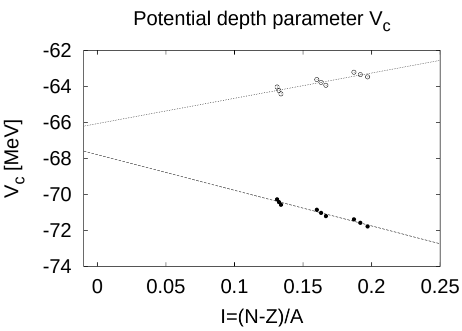

Fig. 76

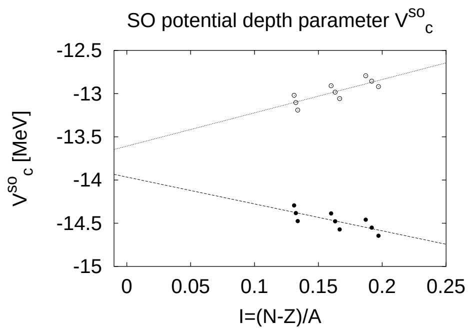

Fig. 77

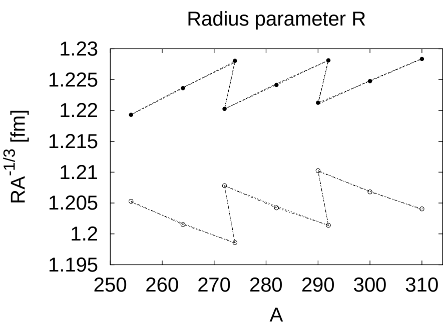

Fig. 78

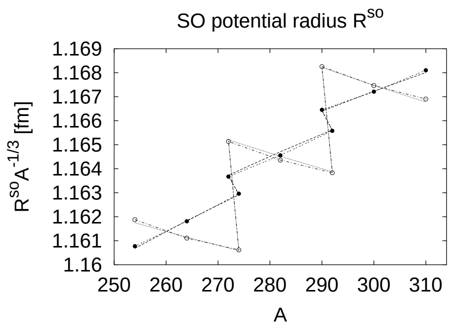

Fig. 79

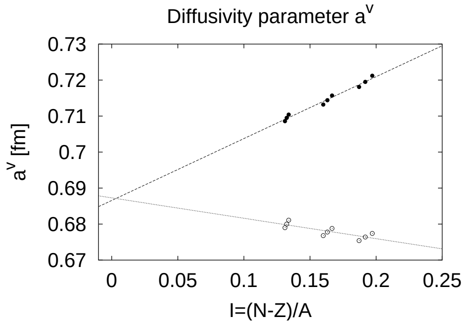

Fig. 80

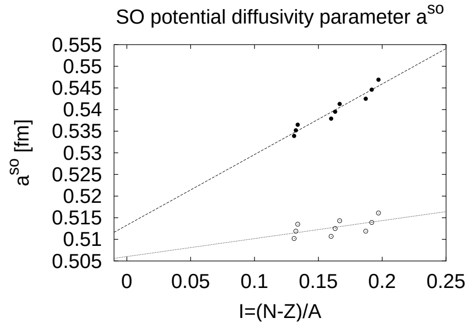

Fig. 81

Effective mass parameters. SLy4

Table 32. Effective mass parameters. SLy4

		Protons			Neutrons		
Z	N	m_c	R^m [fm]	a^m [fm]	m_c	R^m [fm]	a^m [fm]
110	144	0.2788	7.4858	0.4969	0.3033	7.4965	0.5222
110	154	0.2763	7.5739	0.4978	0.3053	7.6046	0.5281
110	164	0.2738	7.6621	0.5000	0.3069	7.7118	0.5350
118	154	0.2775	7.6764	0.4957	0.3020	7.6833	0.5215
118	164	0.2752	7.7603	0.4965	0.3039	7.7862	0.5269
118	174	0.2729	7.8446	0.4983	0.3055	7.8884	0.5333
126	164	0.2763	7.8596	0.4947	0.3008	7.8629	0.5209
126	174	0.2742	7.9399	0.4953	0.3026	7.9612	0.5259
126	184	0.2720	8.0206	0.4969	0.3042	8.0588	0.5319

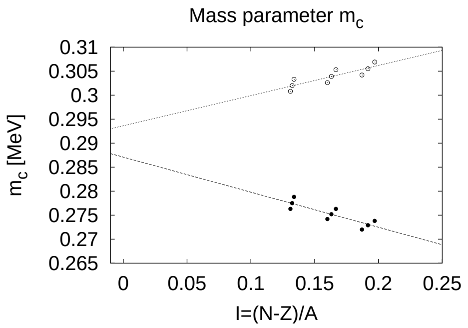

Fig. 82

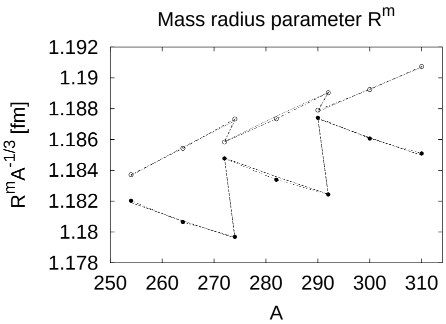

Fig. 83

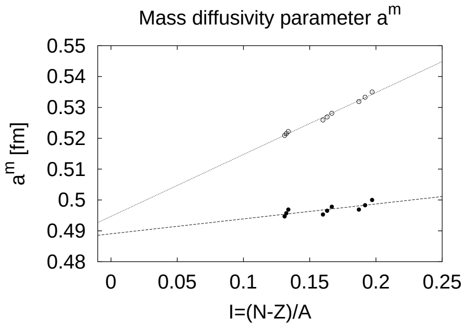

Fig. 84

Coulomb potential parameters. SLy4

Table 33. Coulomb potential parameters. SLy4

Z	N	R^C
110	144	7.6751
110	154	7.7495
110	164	7.8250
118	154	7.8712
118	164	7.9428
118	174	8.0153
126	164	8.0603
126	174	8.1289
126	184	8.1988

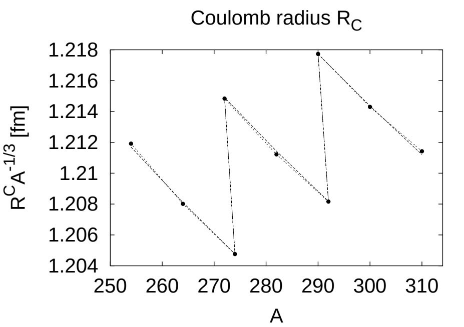

Fig. 85

7. PARAMETERS

Table 34. WS parameters for magic nuclei

		SkIII		SkM*		SLy4*	
Const.	Units	p	n	p	n	p	n
		Central potential					
V_0	MeV	−61.1861	−60.0305	−63.1036	−61.4988	−68.9420	−68.2012
r_0	fm	1.2280	1.2468	1.1992	1.2234	1.1871	1.2036
κ_R^v	–	0.0724	−0.1060	0.1099	−0.0648	0.1647	−0.0938
a_0^v	fm	0.5510	0.5444	0.6932	0.6855	0.7109	0.7166
κ_a^v	–	0.0127	−0.0261	0.0705	−0.0338	0.1357	−0.1853
b_R^v	–	−0.0784	−0.2637	−0.0951	−0.3293	−0.3577	−0.5851
κ^v	–	0.4972	−0.3621	0.4320	−0.4334	0.2520	−0.2141
		Spin-orbit potential					
V_0^{so}	MeV	−13.8418	−13.6712	−16.2629	−16.0346	−15.1082	−14.8981
r_0^{so}	fm	1.1613	1.1671	1.1239	1.1308	1.1276	1.1344
a_0^{so}	fm	0.4432	0.4419	0.5308	0.5306	0.5283	0.5276
b_R^{so}	–	−1.2227	−1.2673	−1.3033	−1.3656	1.1784	−1.2359
κ^{so}	–	0.1024	−0.4168	0.0331	−0.4339	0.0803	−0.3992
κ_R^{so}	–	0.0338	−0.0068	0.0818	0.0229	0.0764	0.0218
κ_a^{so}	–	0.2174	0.0021	0.2573	0.0006	0.2531	−0.0001
		Coulomb potential					
r_0^C	fm	1.217	–	1.194	–	1.197	–
b^C	–	0.620	–	1.103	–	1.166	–
κ^C	–	−0.089	–	−0.102	–	−0.097	–
		Effective nucleon mass					
m_0	MeV/c^2	0.2477	0.2432	0.2183	0.2116	0.3049	0.3085
r_0^m	fm	1.1769	1.1896	1.1321	1.1532	1.1637	1.1549
a_0^m	fm	0.4341	0.4335	0.5102	0.5123	0.5051	0.5046
b_R^m	–	−0.8016	−0.9169	−0.8500	−1.0643	−0.6983	−0.6095
κ^m	–	0.2898	−0.5336	0.3827	−0.7185	−0.3181	0.1223
κ_R^m	–	0.0658	−0.0494	0.1494	−0.0629	−0.0113	0.0846
κ_a^m	–	0.3863	−0.1929	0.5134	−0.2835	0.0305	0.3407
		Nucleon density					
ρ_0	1/fm^3	0.0749	0.0777	0.0808	0.0843	0.0794	0.0828
r_0^ρ	fm	1.1899	1.1900	1.1983	1.1975	1.2026	1.2023
a_0^ρ	fm	0.4630	0.4764	0.5897	0.5995	0.5850	0.5963
b_R^ρ	–	−0.9558	−0.9569	−0.7856	−0.7709	−0.7429	−0.7499
κ^ρ	–	−0.9353	0.6168	−0.9041	0.4910	−0.8835	0.5507
κ_a^ρ	–	−0.4140	0.4047	−0.3478	0.4686	−0.2732	0.4297
κ_R^ρ	–	−0.0625	0.0307	−0.0953	0.0705	−0.0820	0.0472
κ^γ	–	−0.0201	−0.2764	−0.0898	−0.2593	−0.0092	−0.3309
γ_0	–	1.1678	1.3185	1.4997	1.6700	1.5049	1.6727

Table 35. WS parameters for superheavy nuclei

Const.	Units	SkIII p	SkIII n	SkM* p	SkM* n	SLy4* p	SLy4* n
				Central potential			
V_0	MeV	−61.8488	−59.6426	−62.7613	−59.7317	−67.7921	−66.0652
κ^v	–	0.4980	−0.3828	0.4359	−0.4465	0.2920	−0.2126
r_0	fm	1.2289	1.2669	1.2284	1.2733	1.2202	1.2618
κ_R^v	–	0.0455	−0.1054	0.0608	−0.0876	0.0943	−0.1172
b_R^v	–	0.0402	−3.0347	−3.2488	−6.2795	−3.3861	−7.4141
a_0^v	fm	0.5525	0.5242	0.6796	0.6424	0.6866	0.6873
κ_a^v	–	0.5242	0.1049	0.1463	0.1460	0.2504	−0.0824
				Spin-orbit potential			
V_0^{so}	MeV	−12.8367	−12.5653	−14.8103	−14.4159	−13.9657	−13.6080
r_0^{so}	fm	1.2146	1.2253	1.2111	1.2255	1.2087	1.2224
a_0^{so}	fm	0.4289	0.4221	0.5102	0.5044	0.5133	0.5060
b_R^{so}	–	−6.4583	−7.1177	−10.3346	−11.2674	−9.6037	−10.4741
κ^{so}	–	0.3147	−0.2383	0.2150	−0.2813	0.2227	−0.2834
κ_R^{so}	–	−0.0307	−0.0671	−0.0151	−0.0676	−0.0144	−0.0626
κ_a^{so}	–	0.3320	−0.0022	0.3572	−0.0030	0.3182	−0.0031
				Coulomb potential			
r_0^C	fm	1.280	–	1.283	–	1.278	–
b^C	–	−6.722	–	−9.775	–	−8.917	–
κ^C	–	−0.119	–	−0.135	–	−0.126	–
				Effective nucleon mass			
m_0	MeV/c^2	0.2355	0.2279	0.2059	0.1937	0.2871	0.2936
r_0^m	fm	1.2150	1.2412	1.2010	1.2491	1.2351	1.2152
a_0^m	fm	0.4243	0.4155	0.5027	0.4929	0.4890	0.4947
b_R^m	–	−4.4636	−6.3707	−7.9354	−11.4333	−8.6181	−7.0702
κ^m	–	0.4480	−0.4224	0.5214	−0.6511	−0.2544	0.2138
κ_R^m	–	0.0093	−0.0921	0.0554	−0.1309	−0.0685	0.0143
κ_a^m	–	0.4840	−0.0815	0.5649	−0.2524	0.0988	0.4057
				Nucleon density			
ρ_0	1/fm^3	0.0682	0.0726	0.0717	0.0777	0.0716	0.0774
r_0^ρ	fm	1.2249	1.2255	1.2328	1.2321	1.2306	1.2337
a_0^ρ	fm	0.4232	0.4778	0.5397	0.5953	0.5419	0.6010
b_R^ρ	–	−4.8246	−4.2017	−5.3824	−4.3414	−4.4851	−3.9675
κ^ρ	–	−0.7963	0.8572	−0.7885	0.6919	−0.8022	0.7102
κ_a^ρ	–	−0.2403	0.3942	−0.2393	0.4428	−0.1927	0.3559
κ_R^ρ	–	−0.0868	−0.0206	−0.1007	0.0206	−0.0869	0.0037
κ^γ	–	0.3837	−0.3912	0.4097	−0.3409	0.4750	−0.4330
γ_0	–	0.8655	1.3747	1.0008	1.6396	1.0292	1.6673

REFERENCES

[1] M. Brack, C. Guet, H.-B. Håkansson, Phys. Rep. **123** (1985) 275.
[2] M. Brack, R. Bhaduri, *Semiclassical Physics*, Adisson-Wesley, Reading, NY 1997.
[3] I. Zh. Petkov, M.V. Stoitsov, *Nuclear Density Functional Theory*, Oxford Science Publications, Clarendon Press, Oxford, 1991.
[4] J. Bartel, P. Quentin, M. Brack, C. Guet and H.-B. Håkansson, Nucl. Phys. **A386** (1982) 79.
[5] J. Bartel, M. Brack, M. Durand, Nucl. Phys. **A445** (1985) 263.
[6] M. Beiner, H. Flocard, Nguyen van Giai, P. Quentin, Nucl. Phys. **A283** (1975) 29.
[7] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, R. Schaeffer, Nucl. Phys. **A627** (1997) 710, Nucl. Phys. **A635** (1998) 231.
[8] R.D. Woods, A.S. Saxon, Phys. Rev. **95** (1954) 577.
[9] R. Hasse, W.D. Myers, *Geometerical Relationships of Macroscopic Nuclear Physics*, Springer, Berlin, 1988.
[10] Y. Aboussir, J.M. Pearson, A.K. Dutta, F. Tondeur, At. Data and Nucl. Data Tab. **61** (1995) 127.
[11] B. Grammaticos, A. Voros, Ann. Phys. **123** (1979) 359.
[12] J. Bartel, K. Bencheikh, *Nuclear mean fields through self-consistent semiclassical calculations*, to be published in Europ. Phys. Journal A.
[13] M.J. Giannoni, P. Quentin, Phys. Rev. **21** (1980) 2076.
[14] J.C. Slater, Phys. Rev. **81** (1951) 85.
[15] J.W. Negele, D. Vautherin, Phys. Rev. **C5** (1972) 1472.
[16] J. Piperova, D. Samsoen, P. Quentin, K. Bencheikh, J. Bartel, J. Meyer, Nucl. Phys. **A652** (1999) 142.
[17] J. Piperova, private comunication.

Bibliografia

[1] M. Brack, C. Guet, H.-B. Håkansson, Phys. Rep. **123** (1985) 275.
[2] M. Brack, R. Bhaduri, "Semiclassical Physics", Adisson-Wesley Publ. Company, Reading, NY 1997.
[3] I. Zh. Petkov, M.V. Stoitsov, "Nuclear Density Functional Theory", Oxford Science Publications, Clarendon Press, Oxford, 1991.
[4] J. Bartel, P. Quentin, M. Brack, C. Guet and H.-B. Håkansson, Nucl. Phys. **A386** (1982) 79.
[5] J. Bartel, M. Brack, M. Durand, Nucl. Phys. **A445** (1985) 263.
[6] M. Beiner, H. Flocard, Nguyen van Giai, P. Quentin, Nucl. Phys. **A283** (1975) 29.
[7] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, R. Schaeffer, Nucl. Phys. **A627** (1997) 710, Nucl. Phys. **A635** (1998) 231.
[8] R.D. Woods, A.S. Saxon, Phys. Rev. **95** (1954) 577.
[9] R. Hasse, W.D. Myers, *Geometerical Relationships of macroscopic Nuclear Physics*, Springer, Berlin, 1988.
[10] Y. Aboussir, J.M. Pearson, A.K. Dutta, F. Tondeur, At. Data and Nucl. Data Tab. **61** (1995) 127.
[11] B. Grammaticos, A. Voros, Ann. Phys. **123** (1979) 359.
[12] J. Bartel, K. Bencheikh, "Nuclear mean fields through self-consistent semiclassical calculations", to be published in Europ. Phys. Journal A.
[13] M.J. Giannoni, P. Quentin, Phys. Rev. **21** (1980) 2076.
[14] J.C. Slater, Phys. Rev. **81** (1951) 85.
[15] J.W. Negele, D. Vautherin, Phys. Rev. **C5** (1972) 1472.
[16] J. Piperova, D. Samsoen, P. Quentin, K. Bencheikh, J. Bartel,J. Meyer, Nucl. Phys. **A652** (1999) 142.
[17] J. Piperova, private comunication.