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Some geometric constructions of second order connections

Abstract. We determine all natural operators A transforming pairs (Θ, ∇) of second order semiholonomic connections $\Theta : Y \to \overline{J}^2 Y$ and projectable torsion free classical linear connections ∇ on Y into second order semiholonomic connections $A(\Theta, \nabla) : Y \to \overline{J}^2 Y$.

1. Introduction. Denote by $\mathcal{FM}$ the category of fibered manifolds and fiber respecting mappings, by $\mathcal{FM}_m$ the subcategory of fibered manifolds with m-dimensional bases and their fibered maps over local diffeomorphisms and by $\mathcal{FM}_{m,n}$ the subcategory of fibered manifolds with m-dimensional bases, n-dimensional fibres and local fibered diffeomorphisms.

The first jet prolongation $J^1 Y$ of a fibered manifold $Y \to M$ is defined as the bundle of 1-jets of local sections of $Y \to M$. Given an $\mathcal{FM}_m$-map $f : Y_1 \to Y_2$ covering $\underline{f} : M_1 \to M_2$, we have a fibered map $J^1 f : J^1 Y_1 \to J^1 Y_2$ covering f given by $J^1 f(j^1_x \sigma) = j^1_{\underline{f}(x)}(f \circ \sigma \circ \underline{f}^{-1})$, $j^1_x \sigma \in J^1 Y_1$. Using iteration, we obtain the second order nonholonomic prolongation $\tilde{J}^2 Y = J^1(J^1 Y \to M)$. Moreover, the restriction yields the second order semiholonomic prolongation $\overline{J}^2 Y := \{\xi \in \tilde{J}^2 Y \mid \beta_{J^1 Y}(\xi) = J^1 \beta_Y(\xi)\}$, where $\beta_Z : J^1 Z \to Z$ is the bundle projection for any fibered manifold

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$Z \to N$. We have also the second order holonomic prolongation J^2Y, which is the bundle of 2-jets of local sections of $Y \to M$. Clearly, J^2, $\overline{J}^2$ and $\widetilde{J}^2$ are bundle functors $\mathcal{FM}_m \to \mathcal{FM}$ in the sense of [3] that preserve fiber products and we have the obvious inclusions $J^2Y \subset \overline{J}^2Y \subset \tilde{J}^2Y$.

A general connection on a fibered manifold $Y \to M$ is a section $\Gamma : Y \to J^1Y$, which can be also interpreted as a lifting map $Y \times_M TM \to TY$, see [3]. By [1], [2] or [7], it is also useful to study higher order connections, which are defined as sections of higher order jet prolongations of Y. In particular, a second order nonholonomic connection on a fibered manifold $Y \to M$ is a section $\Theta : Y \to \widetilde{J}^2Y$. Such a connection is called semiholonomic or holonomic, if it has values in $\widetilde{J}^2Y$ or J^2Y, respectively. We also recall that a torsion free classical linear connection ∇ on $p : Y \to M$ is called projectable, if there exists a (unique) p-related to ∇ torsion free classical linear connection $\underline{\nabla}$ on M.

In this paper we study the problem how a pair (Θ, ∇) of a second order semiholonomic connection $\Theta : Y \to \overline{J}^2Y$ on $Y \to M$ and a projectable torsion free classical linear connection ∇ on Y can induce canonically a second order semiholonomic connection $A(\Theta, \nabla) : Y \to \overline{J}^2Y$. This problem is reflected in the concept of $\mathcal{FM}_{m,n}$-natural operators $\overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow \overline{J}^2$. In Theorem 1 below we describe all such operators. We also show some applications of our main result. All manifolds and maps are assumed to be infinitely differentiable.

2. Preliminaries. We recall that the general concept of natural operators can be found in [3]. In particular, an $\mathcal{FM}_{m,n}$-natural operator $A : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow \overline{J}^2$ is a system of $\mathcal{FM}_{m,n}$-invariant regular operators (functions)

$$A = A_{Y \to M} : \Gamma(\overline{J}^2Y) \times C_{\tau\text{-}proj}(Y \to M) \to \Gamma(\overline{J}^2Y)$$

for any fibered manifold $Y \to M$, where $\Gamma(\overline{J}^2Y)$ is the set of second order semiholonomic connections on $Y \to M$ and $C_{\tau\text{-}proj}(Y \to M)$ is the set of all projectable torsion free classical linear connections on $Y \to M$. The invariance means that if $\Theta_1 \in \Gamma(\overline{J}^2Y_1)$ and $\Theta_2 \in \Gamma(\overline{J}^2Y_2)$ are f-related by an $\mathcal{FM}_{m,n}$-map $f : Y_1 \to Y_2$ (i.e. $\overline{J}^2f \circ \Theta_1 = \Theta_2 \circ f$) and $\nabla_1 \in C_{\tau\text{-}proj}(Y_1 \to M_1)$ and $\nabla_2 \in C_{\tau\text{-}proj}(Y_2 \to M_2)$ are f-related by the same f, then $A(\Theta_1, \nabla_1)$ and $A(\Theta_2, \nabla_2)$ are f-related. The regularity means that A transforms smoothly parametrized families of pairs of second order semiholonomic connections and projectable torsion free classical linear connections into smoothly parametrized families of second order semiholonomic connections.

Proposition 1. *Second order semiholonomic connections Θ on $Y \to M$ are in bijection with couples (Γ, G) consisting of first order connections Γ on $Y \to M$ and tensor fields $G : Y \to \otimes^2T^*M \otimes VY$.*

Proof. The bijection is given by $(\Gamma, G) \to \Gamma * \Gamma + G$, where $\Gamma * \Gamma = J^1\Gamma \circ \Gamma : Y \to \overline{J}^2 Y$ is the second order semiholonomic Ehresmann prolongation of Γ and the sum operation "+" is the addition in the affine bundle $\overline{J}^2 Y \to J^1 Y$ with the corresponding vector bundle $\otimes^2 T^*M \otimes VY$ over J^1Y. The inverse bijection is given by $\Theta \to (\Gamma, G)$, where Γ is the underlying first order connection of Θ and $G = \Theta - \Gamma * \Gamma$. □

3. The main result. Let Θ be a second order semiholonomic connection on $Y \to M$ and ∇ be a projectable torsion free classical linear connection on $Y \to M$. By Proposition 1 it suffices to classify all $\mathcal{FM}_{m,n}$-natural operators $A_1 : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow J^1$ and $A_2 : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow \otimes^2 T^*B \otimes V$ transforming pairs (Θ, ∇) into first order connections $A_1(\Theta, \nabla)$ on $Y \to M$ and into tensor fields $A_2(\Theta, \nabla) : Y \to \otimes^2 T^*M \otimes VY$, respectively. The definitions of A_1 and A_2 are quite similar to the definition of natural operators $\overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow \overline{J}^2$.

Example 1. Let $\Theta : Y \to \overline{J}^2 Y$ be a second order semiholonomic connection on $Y \to M$ and denote by $(\Gamma^\Theta, G^\Theta)$ the corresponding couple in the sense of Proposition 1. Let ∇ be a projectable torsion free classical linear connection on $Y \to M$. We put $A^o(\Theta, \nabla) = \Gamma^\Theta : Y \to J^1Y$. Then $A^o : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow J^1$ is an $\mathcal{FM}_{m,n}$-natural operator.

Proposition 2. *The operator A^o from Example 1 is the unique $\mathcal{FM}_{m,n}$-natural operator $A_1 : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow J^1$.*

Proof. Let $A_1 : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow J^1$ be an $\mathcal{FM}_{m,n}$-natural operator. It is well known that $J^1Y \to Y$ is an affine bundle with the associated vector bundle $T^*M \otimes VY$. Thus we have the difference operator $\Delta = A_1 - A^o : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow T^*B \otimes V$ given by $\Delta(\Theta, \nabla) = A_1(\Theta, \nabla) - A^o(\Theta, \nabla)$. Then Proposition 2 follows from Lemma 1 below. □

Lemma 1. *Any $\mathcal{FM}_{m,n}$-natural operator $\Delta : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow T^*B \otimes V$ is zero.*

Proof. Any element $\xi \in J^1_0(\mathbf{R}^m \times \mathbf{R}^n)$ is of the form $\xi = j^1_0(x, \sigma(x))$ for some linear map $\sigma : \mathbf{R}^m \to \mathbf{R}^n$. Since a linear $\mathcal{FM}_{m,n}$-map $(x, y - \sigma(x))$ sends $j^1_0(x, \sigma(x))$ into $j^1_0(x, 0)$, $J^1_0(\mathbf{R}^m \times \mathbf{R}^n)$ is the $\mathcal{FM}_{m,n}$-orbit of $\theta^o = j^1_0(x, 0) \in J^1_0(\mathbf{R}^m \times \mathbf{R}^n)$. By the $\mathcal{FM}_{m,n}$-invariance, Δ is determined by the values

$$D(\Gamma, G, \nabla)(0, 0) \in T^*_0\mathbf{R}^m \otimes V_{(0,0)}(\mathbf{R}^m \times \mathbf{R}^n)$$

for all first order connections Γ on $\mathbf{R}^m \times \mathbf{R}^n \to \mathbf{R}^m$ with $\Gamma(0, 0) = \theta^o$, all tensor fields $G : \mathbf{R}^m \times \mathbf{R}^n \to \otimes^2 T^*\mathbf{R}^m \otimes V(\mathbf{R}^m \times \mathbf{R}^n)$ and all projectable torsion free classical linear connections ∇ on $\mathbf{R}^m \times \mathbf{R}^n \to \mathbf{R}^m$. Using the invariance of Δ with respect to the homotheties $\frac{1}{t}\mathrm{id}_{\mathbf{R}^m\times\mathbf{R}^n}$ for $t>0$ and putting $t\to 0$ we deduce that Δ is determined by the value

$$\Delta(\Gamma^o, 0, \nabla^o)(0,0) \in T_0^*\mathbf{R}^m \otimes V_{(0,0)}(\mathbf{R}^m\times\mathbf{R}^n) \tag{1}$$

where Γ^o is the trivial first order connection on $\mathbf{R}^m\times\mathbf{R}^n\to\mathbf{R}^m$ and ∇^o is the usual flat projectable classical linear connection on $\mathbf{R}^m\times\mathbf{R}^n\to\mathbf{R}^m$. Then using the invariance of Δ with respect to fibre homotheties $\mathrm{id}_{\mathbf{R}^m}\times t\mathrm{id}_{\mathbf{R}^n}$ for $t>0$ and putting $t\to 0$ we deduce that the value (1) is zero. That is why, $\Delta=0$. □

So it remains to classify all $\mathcal{FM}_{m,n}$-natural operators $D:\overline{J}^2\times C_{\tau\text{-}proj}\rightsquigarrow\otimes^2T^*B\otimes V$ transforming $\Theta=(\Gamma,G)$ and ∇ into tensor fields $D(\Gamma,G,\nabla):Y\to\otimes^2T^*M\otimes VY$.

Example 2. Let $\Theta=(\Gamma,G)$ be a second order semiholonomic connection on $Y\to M$ and ∇ be a projectable torsion free classical linear connection on $Y\to M$. Take the curvature $C\Gamma=[\Gamma,\Gamma]:Y\to\wedge^2T^*M\otimes VY$ of Γ, see 17.1 in [3]. The correspondence $D_1:\overline{J}^2\times C_{\tau\text{-}proj}\rightsquigarrow\otimes^2T^*B\otimes V$ given by $D_1(\Gamma,G,\nabla)=C\Gamma$ is an $\mathcal{FM}_{m,n}$-natural operator.

Example 3. Denote by $\mathrm{Alt}(G):Y\to\wedge^2T^*M\otimes VY$ the alternation of G. The correspondence $D_2:\overline{J}^2\times C_{\tau\text{-}proj}\rightsquigarrow\otimes^2T^*B\otimes V$ given by $D_2(\Gamma,G,\nabla)=\mathrm{Alt}(G)$ is an $\mathcal{FM}_{m,n}$-natural operator.

Example 4. Denote by $\mathrm{Sym}(G):Y\to S^2T^*M\otimes VY$ the symmetrization of G. The correspondence $D_3:\overline{J}^2\times C_{\tau\text{-}proj}\rightsquigarrow\otimes^2T^*B\otimes V$ given by $D_3(\Gamma,G,\nabla)=\mathrm{Sym}(G)$ is an $\mathcal{FM}_{m,n}$-natural operator.

Example 5. Let (Γ,G,∇) be in question. We have the tangent valued 1-form $\Gamma:Y\to T^*Y\otimes VY$ (the horizontal projection of Γ onto VY). Its covariant derivative $\nabla\Gamma$ can be treated as the tensor field $\nabla\Gamma:Y\to\otimes^2T^*Y\otimes VY$. Composing with the horizontal lifting map $h:Y\to T^*M\otimes TY$ of Γ, we define a tensor field $E(\Gamma,\nabla):Y\to\otimes^2T^*M\otimes VY$. Then the correspondence $D_4:\overline{J}^2\times C_{\tau\text{-}proj}\rightsquigarrow\otimes^2T^*B\otimes V$ given by $D_4(\Gamma,G,\nabla)=E(\Gamma,\nabla)$ is an $\mathcal{FM}_{m,n}$-natural operator.

Remark 1. Of course, we could also take the symmetric and antisymmetric parts of $E(\Gamma,\nabla)$, but such examples will turn the linear combinations of $E(\Gamma,\nabla)$ and $C\Gamma$, see Proposition 3 below.

Proposition 3. *If $m\geq 2$, then all $\mathcal{FM}_{m,n}$-natural operators $D:\overline{J}^2\times C_{\tau\text{-}proj}\rightsquigarrow\otimes^2T^*B\otimes V$ are of the form*

$$D=k_1D_1+k_2D_2+k_3D_3+k_4D_4$$

for (uniquely determined) real numbers k_1, k_2, k_3, k_4. If $m=1$, then $D_1=0$ and $D_2=0$ and we have $D=k_3D_3+k_4D_4$ for some (uniquely determined) $k_3,k_4\in\mathbf{R}$.

Proof. By the above mentioned arguments, D is uniquely determined by the values

(2) $$D(\Gamma, G, \nabla)(0,0) \in \otimes^2 T_0^* \mathbf{R}^m \otimes V_{(0,0)}(\mathbf{R}^m \times \mathbf{R}^n)$$

for all first order connections Γ on $\mathbf{R}^m \times \mathbf{R}^n \to \mathbf{R}^m$ with $\Gamma(0,0) = \theta^o$ and all tensor fields $G : \mathbf{R}^m \times \mathbf{R}^n \to \otimes^2 T^* \mathbf{R}^m \otimes V(\mathbf{R}^m \times \mathbf{R}^n)$ and projectable torsion free classical linear connections ∇ on $\mathbf{R}^m \times \mathbf{R}^n \to \mathbf{R}^m$ such that the identity map $\mathrm{id}_{\mathbf{R}^m \times \mathbf{R}^n}$ is a ∇-normal coordinate system with centre $(0,0)$ (then Christoffell symbols of ∇ in the identity map are zero in $(0,0)$). Using non-linear Peetre theorem, see 19 in [3], and the invariance of D with respect to the homotheties $t\mathrm{id}_{\mathbf{R}^m \times \mathbf{R}^n}$ for $t > 0$ and applying homogeneous function theorem, see 24 in [3], we deduce that the values (2) are of the form

$$D(\Gamma, 0, \nabla^o)(0,0) + D(\Gamma^o, G^o, \nabla^o)(0,0),$$

where Γ^o is the trivial connection and G^o is the constant tensor field such that $G^o(0,0) = G(0,0)$ and ∇^o is the usual flat projectable classical linear connection on $\mathbf{R}^m \times \mathbf{R}^n \to \mathbf{R}^m$. Moreover, if Γ is of the form of the right hand side of (2), then $D(\Gamma, 0, \nabla^o)(0,0)$ is a linear combination of $\frac{\partial}{\partial x^a}\Gamma_j^k(0,0)$ for $a = 1, \ldots, m$ and $\frac{\partial}{\partial y^b}\Gamma_j^k(0,0)$ for $b = 1, \ldots, n$ with real coefficients. By the $\mathcal{FM}_{m,n}$-invariance of D, the map $G^o \to D(\Gamma^o, G^o, \nabla^o)$ can be treated as $\mathrm{GL}(m) \times \mathrm{GL}(n)$-invariant map $\otimes^2(\mathbf{R}^m)^* \otimes \mathbf{R}^n \to \otimes^2(\mathbf{R}^m)^* \otimes \mathbf{R}^n$. It is well known that it is a linear combination of the alternation and symmetrization. Thus replacing D by $D - k_2 D_2 - k_3 D_3$ for some respective real numbers k_2, k_3, we may assume that $D(\Gamma^o, G^o, \nabla^o)(0,0) = 0$. Using the invariance of D with respect to fibre homotheties $\mathrm{id}_{\mathbf{R}^m} \times t\mathrm{id}_{\mathbf{R}^n}$ for all $t > 0$, we deduce that $D(\Gamma, 0, \nabla^o)(0,0)$ is a linear combination of $\frac{\partial}{\partial x^a}\Gamma_j^k(0,0)$ for $a = 1, \ldots, m$. By the invariance of D, the values $D(\Gamma, 0, \nabla^o)(0,0)$ are determined by $\mathrm{GL}(m) \times \mathrm{GL}(n)$-invariant maps $\otimes^2(\mathbf{R}^m)^* \otimes \mathbf{R}^n \to \otimes^2(\mathbf{R}^m)^* \otimes \mathbf{R}^n$. Thus the vector space of all $D(\Gamma, 0, \nabla^o)(0,0)$ is 2-dimensional if $m \geq 2$ (or 1-dimensional if $m = 1$). Then $D = k_1 D_1 + k_4 D_4$ (or $D = k_4 D_4$ if $m = 1$) because of the dimension argument. □

Thus we have proved

Theorem 1. *If $m \geq 2$, then all $\mathcal{FM}_{m,n}$-natural operators $A : \overline{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow \overline{J}^2$ transforming second order semiholonomic connections $\Theta = (\Gamma, G)$ on $Y \to M$ and projectable torsion free classical linear connections ∇ on $Y \to M$ into second order semiholonomic connections on $Y \to M$ are of the form*

$$A(\Theta, \nabla) = (\Gamma, k_1 C\Gamma + k_2 Alt(G) + k_3 Sym(G) + k_4 E(\Gamma, \nabla)), \quad k_i \in \mathbf{R}.$$

If $m = 1$, then $C\Gamma = 0$ and $Alt(G) = 0$ and we have

$$A(\Theta, \nabla) = (\Gamma, k_3 Sym(G) + k_4 E(\Gamma, \nabla))$$

for some uniquely determined $k_3, k_4 \in \mathbf{R}$.

Extracting from Theorem 1 the operators that do not depend on ∇, we have

Corollary 1. *If $m \geq 2$, then all $\mathcal{FM}_{m,n}$-natural operators $A : \overline{J}^2 \rightsquigarrow \overline{J}^2$ transforming second order semiholonomic connections $\Theta = (\Gamma, G)$ on $Y \to M$ into second order semiholonomic connections $A(\Theta)$ on $Y \to M$ are of the form*

$$A(\Theta) = (\Gamma, k_1 C\Gamma + k_2 Alt(G) + k_3 Sym(G)), \quad k_i \in \mathbf{R}.$$

If $m = 1$, then $A(\Theta) = (\Gamma, k_3 Sym(G))$.

For $G = 0$ we reobtain the following result of [6] in another equivalent form.

Corollary 2 ([6])**.** *If $m \geq 2$, then all $\mathcal{FM}_{m,n}$-natural operators $A : J^1 \rightsquigarrow \overline{J}^2$ transforming first order connections Γ on $Y \to M$ into second order semi-holonomic connections $A(\Gamma)$ on $Y \to M$ form the following one-parameter family*

$$A(\Gamma) = (\Gamma, kC\Gamma), \quad k \in \mathbf{R}.$$

If $m = 1$, then $C\Gamma = 0$ and we have $A(\Gamma) = (\Gamma, 0) = \Gamma * \Gamma$.

For second order holonomic connections we have the following version of Proposition 1.

Proposition 4. *Second order holonomic connections $\Theta : Y \to J^2Y$ on $Y \to M$ are in bijection with couples (Γ, G) of first order connections Γ on $Y \to M$ and tensor fields $G : Y \to S^2T^*M \otimes VY$.*

Proof. The bijection is given by $(\Gamma, G) \to C^{(2)}(\Gamma * \Gamma) + G$, where $C^{(2)} : \overline{J}^2Y \to J^2Y$ is the well-known symmetrization of second order semiholonomic jets and the addition "+" is the one of affine bundle $J^2Y \to J^1Y$ with the corresponding associated vector bundle $S^2T^*M \otimes VY$ over J^1Y. The inverse bijection is given by $\Theta \to (\Gamma, G)$, where Γ is the underlying first order connection of Θ and $G = \Theta - C^{(2)}(\Gamma * \Gamma)$. □

Using quite similar methods as above one can show directly

Theorem 2. *All $\mathcal{FM}_{m,n}$-natural operators $A : J^2 \times C_{\tau\text{-}proj} \rightsquigarrow J^2$ transforming second order holonomic connections $\Theta = (\Gamma, G)$ on $Y \to M$ and projectable torsion free classical linear connections ∇ on $Y \to M$ into second order holonomic connections $A(\Theta, \nabla)$ on $Y \to M$ are of the form*

$$A(\Theta, \nabla) = (\Gamma, k_1 G + k_2 Sym(E(\Gamma, \nabla))), \quad k_1, k_2 \in \mathbf{R}.$$

In particular, for the trivial Weil algebra $\mathbf{R}$ we reobtain the following result of [5].

Corollary 3 ([5])**.** *All $\mathcal{FM}_{m,n}$-natural operators $A : J^2 \rightsquigarrow J^2$ transforming second order holonomic connections $\Theta = (\Gamma, G)$ on $Y \to M$ into second order holonomic connection $A(\Theta)$ on $Y \to M$ are of the form*

$$A(\Theta) = (\Gamma, kG), \quad k \in \mathbf{R}.$$

Putting $G = 0$ in Theorem 2 we reobtain the following result of [2].

Corollary 4 ([2])**.** *All $\mathcal{FM}_{m,n}$-natural operators $A : J^1 \times C_{\tau\text{-}proj} \rightsquigarrow J^2$ transforming first order connections Γ on $Y \to M$ and torsion free projectable classical linear connections ∇ on $Y \to M$ into second order holonomic connections $A(\Gamma)$ on $Y \to M$ are of the form*

$$A(\Gamma, \nabla) = (\Gamma, kSym(E(\Gamma, \nabla)), \quad k \in \mathbf{R}.$$

An open problem: It seems that one can also in similar (but more technically complicated) way classify all $\mathcal{FM}_{m,n}$-natural operators $A : \tilde{J}^2 \times C_{\tau\text{-}proj} \rightsquigarrow \tilde{J}^2$ transforming second order nonholonomic connections $\Theta : Y \to \tilde{J}^2 Y$ on $Y \to M$ and torsion free projectable classical linear connections ∇ on $Y \to M$ into second order nonholonomic connections $A(\Theta, \nabla)$ on $Y \to M$. By [1], such $\Theta's$ are in bijection with triples (Γ_1, Γ_2, G) of first order connections Γ_1, Γ_2 on $Y \to M$ and tensor fields $G : Y \to \otimes^2 T^* M \otimes VY$. However, the classification of all above operators A is still an open problem. We inform that in [4] there are described all $\mathcal{FM}_{m,n}$-natural operators $\tilde{J}^2 \rightsquigarrow \tilde{J}^2$ transforming second order nonholonomic connections into themselves.

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