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On the Ehresmann prolongation

Abstract. We determine all natural operators transforming general connections $\Gamma : Y \to J^1Y$ into second order semiholonomic connections $\Sigma : Y \to \overline{J}^2Y$.

Introduction. Let $\mathcal{M}f_m$ be the category of m-dimensional manifolds and local diffeomorphisms, $\mathcal{FM}_m$ be the category of fibered manifolds with m-dimensional bases and fiber respecting mappings with local diffeomorphisms as base maps and $\mathcal{FM}_{m,n}$ be the category of fibered manifolds with m-dimensional bases and n-dimensional fibres provided with locally invertible fiber respecting mappings. Further, let J^rY, $\overline{J}^rY$, $\widetilde{J}^rY$ denote the holonomic, semiholonomic and nonholonomic jet prolongations of a fibered manifold $Y \to M$, respectively, [4], [6].

In this paper we study natural operators transforming connections $\Gamma : Y \to J^1Y$ into second order semiholonomic connections $\Sigma : Y \to \overline{J}^2Y$. In what follows we essentially use the Ehresmann prolongation, [3], due to the property that its values lie always in $\overline{J}^2Y$, [11]. Thus we first recall the definition of Ehresmann prolongation as well as its coordinate expression and further, by a classical procedure, we determine all natural operators in question.

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The motivation for our problem comes from the field of prolongation of principal connections. It turns out that natural operators transforming general connections $\Gamma : Y \to J^1Y$ into higher order connections $\Sigma : Y \to J^rY$ as well as into their semiholonomic and nonholonomic analogues seem to be a useful tool for the description of principal connections on higher order principal prolongations W^rP of a principal bundle $P \to M$, [6], [10]. We recall that the classification of all gauge natural operators transforming a connection on a principal bundle $P \to M$ and a linear symmetric connection on the base manifold M into a connection on the first order principal prolongation W^1P is known and can be found in [9]. But for higher orders the situation is much more complicated and such description still remains an open question even due to the fact that among higher order prolongations we have to distinguish the holonomic, semiholonomic and nonholonomic case.

Finally let us mention that the procedure of finding natural operators used in this paper is classical and its complete description can be found in [6]. A good example of its application can be also found in [8]. But this method becomes rather technically complicated when we try to determine operators transforming general connections $\Gamma : Y \to J^1Y$ into higher order connections $\Sigma : Y \to J^rY$ (or their semiholonomic or nonholonomic analogue) and thus these operators remain unknown for $r > 2$.

1. Foundations. Let $p : Y \to M$ be a fibered manifold. By (x^i), $i = 1, \dots, m$ we denote the local coordinates on M and by (x^i, y^p), $i = 1, \dots, m$, $p = 1, \dots, n$ the local coordinates on Y. Given a fibered manifold $p : Y \to M$, let us denote by $J^rY \to M$ its r-th jet prolongation, that is the space of r-jets of local sections $M \to Y$. In what follows, J^rY will be called the r-th holonomic prolongation of Y. Recall that the r-th nonholonomic prolongation $\widetilde{J}^rY$ of Y is defined by the iteration

$$\widetilde{J}^1Y = J^1Y, \qquad \widetilde{J}^rY = J^1\left(\widetilde{J}^{r-1}Y \to M\right).$$

Clearly, we have an inclusion $J^rY \subset \widetilde{J}^rY$ given by $j^r_x\gamma \mapsto j^1_x(j^{r-1}\gamma)$. Further, the r-th semiholonomic prolongation $\overline{J}^rY \subset \widetilde{J}^rY$ is defined by the following induction. First we denote by $\beta_1 = \beta_Y$ the projection $J^1Y \to Y$ and by $\beta_r = \beta_{\widetilde{J}^{r-1}Y}$ the projection $\widetilde{J}^rY = J^1\widetilde{J}^{r-1}Y \to \widetilde{J}^{r-1}Y$, $r = 2, 3, \dots$. Then we set $\overline{J}^1Y = J^1Y$ and assume we have $\overline{J}^{r-1}Y \subset \widetilde{J}^{r-1}Y$ such that the restriction of the projection $\beta_{r-1} : \widetilde{J}^{r-1}Y \to \widetilde{J}^{r-2}Y$ maps $\overline{J}^{r-1}Y$ into $\overline{J}^{r-2}Y$. Then we can construct $J^1\beta_{r-1} : J^1\overline{J}^{r-1}Y \to J^1\overline{J}^{r-2}Y$ and define

$$\overline{J}^rY = \left\{A \in J^1\overline{J}^{r-1}Y;\ \beta_r(A) \in \overline{J}^{r-1}Y\right\}.$$

We recall that the induced coordinates on the holonomic prolongation J^rY are given by (x^i, y^p_α), where α is a multiindex of range m satisfying $|\alpha| \le r$.

Clearly, the coordinates y^p_α on J^rY are characterized by the full symmetry in the indices α. Having the nonholonomic prolongation $\widetilde{J}^rY$ constructed by the iteration, we define the local coordinates inductively as follows:

1) Suppose that the coordinates on $\widetilde{J}^{r-1}Y$ are of the form

$$\left(x^i, y^p_{k_1\dots k_{r-1}}\right),\ k_1,\dots,k_{r-1}=0,1,\dots,m.$$

2) We define the induced coordinates on $\widetilde{J}^rY$ by

$$\left(x^i, y^p_{k_1\dots k_{r-1}0}=y^p_{k_1\dots k_{r-1}}, y^p_{k_1\dots k_{r-1}i}=\frac{\partial}{\partial x^i}y^p_{k_1\dots k_{r-1}}\right).$$

It remains to describe coordinates on the semiholonomic prolongation $\overline{J}^rY$. Let $(k_1,\dots,k_r)$, $k_1,\dots,k_r=0,1,\dots,m$ be a sequence of indices and denote by $\langle k_1,\dots,k_s\rangle$, $s\le r$ the sequence of non-zero indices in $(k_1,\dots,k_r)$ respecting the order. Obviously, the point $(x^i, y^p_{k_1\dots k_r})\in\widetilde{J}^rY$ belongs to $\overline{J}^rY$ if and only if $y^p_{k_1\dots k_r}=y^p_{l_1\dots l_r}$ whenever $\langle k_1,\dots,k_r\rangle=\langle l_1,\dots,l_r\rangle$. Thus the coordinates on $\overline{J}^rY$ are $x^i, y^p_{i_1\dots i_s}$, $s=0,\dots,r$, which are arbitrary in the subscripts. Further, the elements of $J^rY\subset\overline{J}^rY$ are characterized by the full symmetry in all subscripts.

Now we recall that general connection on a fibered manifold $p:Y\to M$ can be defined as a section $\Gamma:Y\to J^1Y$ of the first jet prolongation $J^1Y\to Y$ or, equivalently, as a lifting map $\Gamma: Y\times_M TM\to TY$. In local coordinates, a general connection Γ is given by

$$\mathrm{d}y^p=F^p_i(x,y)\mathrm{d}x^i,$$

where $F^p_i(x,y)$ are smooth functions.

Further, let $\widetilde{J}^rY\to M$ be the r-th nonholonomic jet prolongation of a fibered manifold $p:Y\to M$. In general, an *r-th order nonholonomic connection* on Y is a section $\Gamma:Y\to\widetilde{J}^rY$. Such a connection is called semiholonomic or holonomic, if it has values in $\overline{J}^rY$ or in J^rY, respectively.

Let us recall that by curvature $C(\Gamma)$ of a connection Γ on $Y\to M$ we understand a map

$$C(\Gamma): Y\times_M\wedge^2TM\to VY$$

defined by

$$C(\Gamma)(y,X,Z)=\left(\Gamma([X,Z])-[\Gamma X,\Gamma Z]\right)(y)$$

for any vector fields X,Z on M and $y\in Y$, where ΓX means the Γ-lift of the vector field X and $[\ ,\ ]$ denotes the Lie bracket. Using the above notation for the local coordinates, the curvature $C(\Gamma)$ of a connection Γ has the following coordinate expression:

$$\mathrm{d}y^p\ =\left(\frac{\partial F^p_j}{\partial x^i}+\frac{\partial F^p_j}{\partial y^q}F^q_i\right)\mathrm{d}x^i\wedge\mathrm{d}x^j.$$

2. Ehresmann prolongation. Given two higher order connections $\Gamma : Y \to \widetilde{J}^r Y$ and $\overline{\Gamma} : Y \to \widetilde{J}^s Y$, the product of Γ and $\overline{\Gamma}$ is the $(r+s)$-th order connection $\Gamma * \overline{\Gamma} : Y \to \widetilde{J}^{r+s} Y$ defined by

$$\Gamma * \overline{\Gamma} = \widetilde{J}^s \Gamma \circ \overline{\Gamma}.$$

As an example we show the coordinate expression of an arbitrary nonholonomic second order connection and of the product of two first order connections. The coordinate form of $\Sigma : Y \to \widetilde{J}^2 Y$ is

$$y_i^p = F_i^p(x,y), \quad y_{0i}^p = G_i^p(x,y), \quad y_{ij}^p = H_{ij}^p(x,y),$$

where F, G, H are arbitrary smooth functions. Further, if the coordinate expressions of two first order connections $\Gamma, \overline{\Gamma} : Y \to J^1 Y$ are

$$\Gamma : \quad y_i^p = F_i^p(x,y), \qquad \overline{\Gamma} : \quad y_i^p = G_i^p(x,y),$$

then the second order connection $\Gamma * \overline{\Gamma} : Y \to \widetilde{J}^2 Y$ has equations

$$\begin{aligned} y_i^p &= F_i^p \\ y_{0i}^p &= G_i^p \\ y_{ij}^p &= \frac{\partial F_i^p}{\partial x^j} + \frac{\partial F_i^p}{\partial y^q} G_j^q. \end{aligned}$$

For two first order connections Γ and $\overline{\Gamma}$ the product $\Gamma * \overline{\Gamma} : Y \to \widetilde{J}^2 Y$ is semiholonomic if and only if $\Gamma = \overline{\Gamma}$, [5], [11].

Considering a connection $\Gamma : Y \to J^1 Y$, we can define an r-th order connection $\Gamma^{(r-1)} : Y \to \widetilde{J}^r Y$ by

$$\Gamma^{(1)} := \Gamma * \Gamma = J^1 \Gamma \circ \Gamma, \qquad \Gamma^{(r-1)} := \Gamma^{(r-2)} * \Gamma = J^1 \Gamma^{(r-2)} \circ \Gamma.$$

The connection $\Gamma^{(r-1)}$ is called the $(r-1)$-st prolongation of Γ in the sense of Ehresmann, shortly $(r-1)$-st Ehresmann prolongation.

As an example we recall the coordinate expression of $\Gamma^{(1)}$. Let $y_i^p = F_i^p(x,y)$ be the coordinate expression of a connection $\Gamma : Y \to J^1 Y$. Then the connection $\Gamma^{(1)} = \Gamma * \Gamma : Y \to \overline{J}^2 Y$ has equations

$$y_i^p = F_i^p, \quad y_{ij}^p = \frac{\partial F_i^p}{\partial x^j} + \frac{\partial F_i^p}{\partial y^q} F_j^q.$$

For our purposes the following proposition is essential as it explains the use of Ehresmann prolongation for the semiholonomic version of the operator in question:

Proposition 2.1. *The values of $\Gamma^{(r-1)}$ lie in the semiholonomic prolongation $\overline{J}^r Y$ and $\Gamma^{(r-1)}$ is holonomic if and only if Γ is curvature free.*

For the proof see [5], [11].

For second order nonholonomic connections we have the following identification only, [6]:

Proposition 2.2. *Second order nonholonomic connections on $Y \to M$ are in bijection with triples $(\Gamma, \overline{\Gamma}, \Sigma)$, where $\Gamma, \overline{\Gamma} : Y \to J^1Y$ are first order connections on $Y \to M$ and $\Sigma : Y \to VY \otimes \overset{2}{\otimes} T^*M$ is a section.*

3. The main result. In this section we find all natural operators transforming a connection $\Gamma : Y \to J^1Y$ into a second order semiholonomic connection $\Sigma : Y \to \overline{J}^2Y$.

Given the local coordinates $(x^i, y^p, y^p_i, y^p_{ij})$ on $\overline{J}^2Y$, we have a natural map $e : \overline{J}^2Y \to \overline{J}^2Y$ with the coordinate expression

$$y^p_i = y^p_i, \qquad y^p_{ij} = y^p_{ji},$$

see [6].

I. Kolář and M. Modugno proved in [7]

Proposition 3.1. *All natural transformations $\overline{J}^2 \to \overline{J}^2$ form a one parameter family*

$$X \mapsto kX + (1-k)e(X), \qquad k \in \mathbb{R}.$$

Now we are ready to formulate

Proposition 3.2. *All natural operators transforming first order connection $\Gamma : Y \to J^1Y$ into second order semiholonomic connection $Y \to \overline{J}^2Y$ form a one parameter family*

$$\Gamma \mapsto k \cdot (\Gamma * \Gamma) + (1-k) \cdot e(\Gamma * \Gamma), \qquad k \in \mathbb{R}. \tag{1}$$

Proof. The proof has three steps typical for this method.

Step 1. First we determine the first order operators. According to the general theory, [6], it is enough to find all $G^2_{m,n}$-equivariant maps $(J^1J^1)_0 \to (\overline{J}^2)_0$ of the standard fibers, where $G^2_{m,n}$ is the group of all r-jets at $0 \in \mathbb{R}^{m+n}$ of fibered manifold isomorphisms $f : (\mathbb{R}^{m+n} \to \mathbb{R}^m) \to (\mathbb{R}^{m+n} \to \mathbb{R}^m)$ satisfying $f(0) = 0$. This leads to the $G^2_{m,n}$-equivariant maps

$$f : S_1 \to Z$$

over $0 \in \mathbb{R}^{m+n}$, where $S_1 := J^1_0(J^1(\mathbb{R}^{m+n} \to \mathbb{R}^m) \to \mathbb{R}^{m+n})$ and $Z := \overline{J}^2_0(\mathbb{R}^{m+n} \to \mathbb{R}^m)$. Let us denote the local coordinates as follows:

$$S_1 : \left(y^p_i = \frac{\partial y^p}{\partial x^i}, y^p_{ij} = \frac{\partial y^p_i}{\partial x^j}, y^p_{iq} = \frac{\partial y^p_i}{\partial y^q} \right)$$

$$Z : \left(Z^p_i, Z^p_{ij} \right)$$

We have to determine all $G^2_{m,n}$-equivariant maps

$$Z^p_i = f^p_i(y^p_i, y^p_{ij}, y^p_{iq})$$

$$Z^p_{ij} = f^p_{ij}(y^p_i, y^p_{ij}, y^p_{iq}),$$

which express the coordinate form of the natural operators in question.

Let $L^r(p) \subset L^r_{m,n} := J^r_0(\mathbb{R}^m, \mathbb{R}^n)_0$ be the subgroup of the r-jets of the diffeomorphisms $x^i = x^i(x),\ y^p = y^p(x,y)$ preserving the fibration $\mathbb{R}^{m+n} \to \mathbb{R}^m$. In particular, the coordinates on $L^2(p)$, which correspond to the values of the partial derivatives of x^i and y^p at the origin are

$$a^i_j,\ a^i_{jk},\ a^p_i,\ a^p_q,\ a^p_{ij},\ a^p_{qi},\ a^p_{qr}$$

and the coordinates of the inverse elements will be denoted by a tilde. Evaluating the effect of the isomorphisms in $\mathcal{FM}_{m,n}$ and passing to 2-jets, we obtain the following transformation laws on S_1, i.e. the action of the group $L^2(p)$ on S_1, see [6]:

$$\overline{y}^p_i = a^p_q y^q_j \widetilde{a}^j_i + a^p_j \widetilde{a}^j_i \tag{2}$$

$$\overline{y}^p_{iq} = a^p_r y^r_{js} \widetilde{a}^s_q \widetilde{a}^j_i + a^p_{rs} y^r_j \widetilde{a}^s_q \widetilde{a}^j_i + a^p_{rj} \widetilde{a}^r_q \widetilde{a}^j_i \tag{3}$$

$$\begin{aligned}\overline{y}^p_{ij} &= a^p_q y^q_{kl} \widetilde{a}^k_i \widetilde{a}^l_j + a^p_q y^q_{kr} \widetilde{a}^r_j \widetilde{a}^k_i + a^p_{ql} y^q_k \widetilde{a}^k_i \widetilde{a}^l_j + a^p_{qr} y^q_k \widetilde{a}^r_j \widetilde{a}^k_i \\ &\quad + a^p_q y^q_k \widetilde{a}^k_{ij} + a^p_{kl} \widetilde{a}^k_i \widetilde{a}^l_j + a^p_{kq} \widetilde{a}^q_j \widetilde{a}^k_i + a^p_k \widetilde{a}^k_{ij}.\end{aligned} \tag{4}$$

Further, the transformation law of the induced coordinates $Z^p_i,\ Z^p_{ij}$ is of the form

$$\overline{Z}^p_i = a^p_q Z^q_j \widetilde{a}^j_i + a^p_j \widetilde{a}^j_i \tag{5}$$

$$\begin{aligned}\overline{Z}^p_{ij} &= a^p_q Z^q_{kl} \widetilde{a}^k_i \widetilde{a}^l_j + a^p_{qr} Z^r_k Z^q_l \widetilde{a}^k_j \widetilde{a}^l_i + a^p_{qk} Z^q_l \widetilde{a}^l_j \widetilde{a}^k_i \\ &\quad + a^p_{qk} Z^q_l \widetilde{a}^k_j \widetilde{a}^l_i + a^p_q Z^q_l \widetilde{a}^l_{ij} + a^p_{kl} \widetilde{a}^l_j \widetilde{a}^k_i + a^p_k \widetilde{a}^k_{ij}.\end{aligned} \tag{6}$$

The equivariance of f^p_i and f^p_{ij} with respect to the base homotheties determined by $\widetilde{a}^i_j = k\delta^i_j$, $a^p_q = \delta^p_q$, yields

$$\begin{aligned} k f^p_i &= f^p_i(k y^p_i, k^2 y^p_{ij}, k y^p_{iq}) \\ k^2 f^p_{ij} &= f^p_{ij}(k y^p_i, k^2 y^p_{ij}, k y^p_{iq}), \end{aligned}$$

while the equivariance with respect to the fiber homotheties determined by $\widetilde{a}^i_j = \delta^i_j,\ a^p_q = k\delta^p_q$, gives

$$\begin{aligned} k f^p_i &= f^p_i(k y^p_i, k y^p_{ij}, y^p_{iq}) \\ k f^p_{ij} &= f^p_{ij}(k y^p_i, k y^p_{ij}, y^p_{iq}). \end{aligned}$$

By the homogeneous function theorem, [6], f^p_i are linear in y^p_i and do not depend on y^p_{ij} and y^p_{iq}, while f^p_{ij} are linear in y^p_{ij} and bilinear in y^p_{iq} and y^p_i. So according to generalized invariant tensor theorem, [6], we have

$$\begin{aligned} f^p_i &= a y^p_i \\ f^p_{ij} &= k_1 y^p_{ij} + k_2 y^p_{ji} + k_3 y^p_{iq} y^q_j + k_4 y^p_{jq} y^q_i + k_5 y^q_{iq} y^p_j + k_6 y^q_{jq} y^p_i, \end{aligned}$$

where $a, k_1, \ldots, k_6$ are real constants. If we consider the full equivariance with respect to the subgroup determined by $a^i_j = \delta^i_j$, $a^p_q = \delta^p_q$, we obtain

$$k_1 + k_2 = 1, \quad k_3 + k_4 = 1, \quad k_1 = k_3 \text{ and } k_2 = k_4.$$

This implies, that the coordinate expression of f is of the form

$$Z^p_i = y^p_i, \qquad Z^p_{ij} = k \cdot (y^p_{ij} + y^p_{iq} y^q_j) + (1-k) \cdot (y^p_{ji} + y^p_{jq} y^q_i), \quad k \in \mathbb{R},$$

which corresponds to (1).

Step 2. Assume we have an r-th order operator $J^1 \rightsquigarrow \overline{J}^2$. It corresponds to a G^{r+1}_{m+n}-equivariant map from the standard fibre $S_r := J^r_0(J^1(\mathbb{R}^{m+n} \to \mathbb{R}^m) \to \mathbb{R}^{m+n})$ into Z. Denote by $y^p_{i\alpha\beta}$ the partial derivative of y^p_i with respect to a multiindex α in x^i and β in y^p. Then the associated map of our operator has the form

$$Z^p_i = f^p_i(y_{i\alpha\beta}), \qquad Z^p_{ij} = f^p_{ij}(y_{i\alpha\beta})$$

where $|\alpha| + |\beta| \leq r$. Using the base homotheties we obtain

$$k f^p_i = f^p_i(k^{|\alpha|+1} y^p_{i\alpha\beta}).$$

Hence f^p_i are linear in y^p_i and independent of any variables $|\alpha| \geq 1$. The fiber homotheties imply that f^p_i are linear in coordinates with $|\beta| = 0$ and independent of the coordinates with $|\beta| \geq 1$. Thus $f^p_i = a y^p_i$. For f^p_{ij} the base homotheties yield

$$k^2 f^p_{ij} = f^p_{ij}(k^{|\alpha|+1} y^p_{i\alpha\beta}) \tag{7}$$

so that f^p_{ij} are bilinear in coordinates with $|\alpha| = 0$ and linear in coordinates with $|\alpha| = 1$. The fiber homotheties imply

$$k f^p_{ij} = f^p_{ij}(k^{1-|\beta|} y_{i\alpha\beta}). \tag{8}$$

Combining (7) and (8) we deduce that f^p_{ij} are independent of $y_{i\alpha\beta}$ with $|\alpha| + |\beta| > 1$, which is the operator of order 1.

Step 3. According to the finite order theorem, [6], every natural operator of our type has finite order.

This completes the proof. □

Remark. In other words, all natural operators from Proposition 3.2 can be obtained from the Ehresmann prolongation $\Gamma * \Gamma$ by applying all natural transformations $\overline{J}^2 \to \overline{J}^2$ from Proposition 3.1.

Clearly, (1) can be written also as

$$\Gamma \mapsto (\Gamma * \Gamma) + t(\Gamma * \Gamma - e(\Gamma * \Gamma)), \qquad t \in \mathbb{R}. \tag{9}$$

We recall, that the difference tensor $\delta(U)$ of a semiholonomic 2-jet $U \subset \overline{J}^2 Y$ is the map $\delta : \overline{J}^2 Y \to VY \otimes \wedge^2 T^* M$ defined by $\delta(U) := U - e(U)$, in local coordinates

$$\delta(y^p_{ij}) = y^p_{ij} - y^p_{ji}.$$

Obviously, in our situation δ corresponds to the term $\Gamma * \Gamma - e(\Gamma * \Gamma)$ in (9).

Further, we can consider the connection $\Gamma * \Gamma$ as a section $Y \to \overline{J}^2 Y$. The bundle $\overline{J}^2 Y \to J^1 Y$ is an affine bundle with the associated vector bundle

$$VY \otimes \overset{2}{\otimes} T^* M = (VY \otimes S^2 T^* M) \oplus (VY \otimes \wedge^2 T^* M),$$

where the second part is determined by the values of the difference tensor δ. The coordinate expression of (9) implies, that if Γ is curvature free, then the difference tensor is zero and thus the associated vector bundle is reduced to the symmetric part. This corresponds to the subbundle $J^2 Y \to J^1 Y$. We showed above, that if Γ is curvature free, the connection $\Gamma * \Gamma$ has values in holonomic jet prolongation $J^2 Y$, see also [11].

We remark that A. Cabras and I. Kolář have systematically studied the prolongation of second order connections to vertical Weil bundles $V^A Y \to M$. Further, M. Doupovec and W. M. Mikulski [1] have characterized all bundle functors F on $\mathcal{FM}_m$, which admit natural operators transforming higher order connections on $Y \to M$ into higher order connections on $FY \to M$. The same authors have also introduced the prolongation of higher order connections to higher order jet bundles by means of some auxiliary linear connection Δ on the base manifold, [2].

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