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Canonical vector valued 1-forms on higher order adapted frame bundles over category of fibered squares

Abstract. Let Y be a fibered square of dimension (m_1, m_2, n_1, n_2). Let V be a finite dimensional vector space over $\mathbb{R}$. We describe all $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical V-valued 1-form $\Theta\colon TP^r_A(Y) \to V$ on the r-th order adapted frame bundle $P^r_A(Y)$.

A fibered square (or fibered-fibered manifold) is any commutative diagram

$$\begin{array}{ccc} Y & \xrightarrow{\pi} & X \\ {\scriptstyle q}\downarrow & & \downarrow{\scriptstyle p} \\ N & \xrightarrow{\pi_0} & M \end{array} \tag{1}$$

where maps π, π_0, q, p are surjective submersions and induced map $Y \to X \times_M N$, $y \mapsto (\pi(y), q(y))$ is a surjective submersion. We will denote a fibered square (1) by Y in short, [3], [5].

A fibered square (1) has dimension (m_1, m_2, n_1, n_2), if $\dim Y = m_1+m_2+n_1+n_2$, $\dim X = m_1+m_2$, $\dim N = m_1+n_1$, $\dim M = m_1$. For two fibered squares Y_1, Y_2 of the same dimension (m_1, m_2, n_1, n_2), a fibered squares morphism $f\colon Y_1 \to Y_2$ is quadruple of local diffeomorphisms $f\colon Y_1 \to Y_2$,

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$f_1\colon X_1 \to X_2$, $f_2\colon N_1 \to N_2$, $f_0\colon M_1 \to M_2$ such that all squares of the cube in question are commutative.

All fibered squares of given dimension (m_1, m_2, n_1, n_2) and their morphisms form a category which we denote by $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$.

Every object from the category $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$ is locally isomorphic to the standard fibered square

$$
\begin{array}{ccc}
\mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} & \longrightarrow & \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \\
\downarrow & & \downarrow \\
\mathbb{R}^{m_1} \times \mathbb{R}^{n_1} & \longrightarrow & \mathbb{R}^{m_1}
\end{array} \tag{2}
$$

which we denote by $\mathbb{R}^{m_1,m_2,n_1,n_2}$, where arrows are obvious projections.

Let Y be a fibered square (1) of dimension (m_1, m_2, n_1, n_2). We define the r-th order adapted frame bundle

$$
\begin{aligned}
P^r_A(Y) = \{j^r_{(0,0,0,0)}\varphi \mid \varphi\colon \mathbb{R}^{m_1,m_2,n_1,n_2} \to Y \text{ is}& \\
\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}\text{ - morphism}\}&
\end{aligned} \tag{3}
$$

over Y with the projection $\beta\colon P^r_A(Y) \to Y$, $\beta(j^r_{(0,0,0,0)}\varphi) = \varphi(0,0,0,0)$. The adapted frame bundle $P^r_A(Y)$ is a principal bundle with Lie group $G^r_{m_1,m_2,n_1,n_2} = P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})_{(0,0,0,0)}$ (with multiplication given by the composition of jets) acting on the right on $P^r_A(Y)$ by composition of jets. Every $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-morphism $\Phi\colon Y_1 \to Y_2$ induces a local diffeomorphism $P^r_A\Phi\colon P^r_A(Y_1) \to P^r_A(Y_2)$ given by $P^r_A\Phi(j^r_{(0,0,0,0)}\varphi) = j^r_{(0,0,0,0)}(\Phi \circ \varphi)$, [1], [4].

Definition 1. Let V be a finite dimensional vector space over $\mathbb{R}$. A $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical V-valued 1-form Θ on P^r_A is any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-invariant family $\Theta = \{\Theta_Y\}$ of V-valued 1-forms $\Theta_Y\colon TP^r_A(Y) \to V$ on $P^r_A(Y)$ for any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y, [2], [4].

The invariance of canonical 1-form Θ means that two V-valued forms Θ_{Y_1} and Θ_{Y_2} are $P^r_A\Phi$-related (that is $P^r_A\Phi^*\Theta_{Y_2} = \Theta_{Y_1}$, where $P^r_A\Phi^*\Theta_{Y_2} = \Theta_{Y_2} \circ TP^r_A\Phi$) for any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-morphism $\Phi\colon Y_1 \to Y_2$.

Example 1. For every $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y we define $\mathbb{R}^{m_1+m_2+n_1+n_2}$-valued 1-form θ_Y on $P^1_A(Y)$ as follows. Consider the projection $\beta\colon P^1_A(Y) \to Y$ given by $\beta(j^1_{(0,0,0,0)}\varphi) = \varphi(0,0,0,0)$, an element $u = j^1_{(0,0,0,0)}\psi \in P^1_A(Y)$ and a tangent vector $W = j^1_0 c \in T_u P^1_A(Y)$. We define the form θ_Y by

$$
\begin{aligned}
\theta_Y(W) &= u^{-1} \circ T\beta(W) \\
&= j^1_0(\psi^{-1} \circ \beta \circ c) \in T_{(0,0,0,0)}\mathbb{R}^{m_1+m_2+n_1+n_2} = \mathbb{R}^{m_1+m_2+n_1+n_2}.
\end{aligned} \tag{4}
$$

Obviously, $\theta = \{\theta_Y\}$ is a $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical 1-form on P^1_A.

A vector field W on Y is projectable-projectable on $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object (1), if there exists vector fields W_1 on X and W_2 on N and W_0 on

M such that W, W_1 are π-related and W, W_2 are q-related and W_1, W_0 are p-related and W_2, W_0 are π_0-related, [5].

We therefore see that vector field W on Y is projectable-projectable on $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object (1) if and only if the flow $\{\Phi_t\}$ of vector field W is formed by local $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-maps.

The space of all projectable-projectable vector fields on $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y will be denoted by $\mathfrak{X}_{proj\text{-}proj}(Y)$. It is Lie subalgebra of Lie algebra $\mathfrak{X}(Y)$ of all vector fields on Y.

For projectable-projectable vector field $W \in \mathfrak{X}_{proj\text{-}proj}(Y)$ the flow lifting $\mathcal{P}^r_A W$ is vector field on $P^r_A(Y)$ such that if $\{\Phi_t\}$ is the flow of field W, then $\{P^r_A(\Phi_t)\}$ is the flow of field $\mathcal{P}^r_A W$. (Since Φ_t are $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-maps, we can apply functor P^r_A to Φ_t).

To present a general example of a $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical V-valued 1-form on P^r_A we need the following lemma, which is an obvious modification of the known fact for usual manifolds.

Lemma 1. *Let Y be a fibered square* (1) *from the category $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$. Then any vector $w \in T_v P^r_A(Y)$, where $v \in (P^r_A(Y))_y$, $y \in Y$, is of the form $w = \mathcal{P}^r_A W_v$ for any projectable-projectable vector field $W \in \mathfrak{X}_{proj\text{-}proj}(Y)$, where $\mathcal{P}^r_A W \in \mathfrak{X}(P^r_A(Y))$ is the flow lifting of field W to $P^r_A(Y)$. Moreover $j^r_y W$ is uniquely determined.*

Proof. We can assume that $Y = \mathbb{R}^{m_1,m_2,n_1,n_2}$ and $y = (0,0,0,0) \in \mathbb{R}^{m_1+m_2+n_1+n_2}$. Since $P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$ is obviously a principal subbundle of the r-th order frame bundle $P^r(\mathbb{R}^{m_1+m_2+n_1+n_2})$, by the well-known manifolds version of Lemma 1, we find $W \in \mathfrak{X}(\mathbb{R}^{m_1+m_2+n_1+n_2})$ such that $w = \mathcal{P}^r W_v$ and $j^r_{(0,0,0,0)} W$ is uniquely determined, where $\mathcal{P}^r W$ is a vector field on $P^r(\mathbb{R}^{m_1+m_2+n_1+n_2})$ being a flow lifting of vector field W and $v \in P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$.

For a projectable-projectable vector field $\widetilde{W} \in \mathfrak{X}_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ the vector $\mathcal{P}^r \widetilde{W}_v \in T_v P^r(\mathbb{R}^{m_1+m_2+n_1+n_2})$ is tangent to $P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$ at the point v. On the other hand, the dimension of space $P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$ and the dimension of space of r-jets $j^r_{(0,0,0,0)}\widetilde{W}$ of projectable-projectable vector fields $\widetilde{W} \in \mathfrak{X}_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ are equal. Then Lemma 1 follows from dimension equality, since flow operators are linear. □

Example 2. Let

$$\lambda\colon J^{r-1}_{(0,0,0,0)}(T_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2})) \to V \tag{5}$$

be a $\mathbb{R}$-linear map, where $J^{r-1}_{(0,0,0,0)}(T_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2}))$ is the vector space of all $(r-1)$-jets $j^{r-1}_{(0,0,0,0)}W$ at point $(0,0,0,0) \in \mathbb{R}^{m_1+m_2+n_1+n_2}$ of projectable-projectable vector fields $W \in \mathfrak{X}_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2})$. Given a fibered square Y, (1), from the category $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$ we define V-valued 1-form $\Theta^\lambda_Y\colon TP^r_A(Y) \to V$ on $P^r_A(Y)$ as follows. Let $w \in T_vP^r_A(Y)$, where $v = j^r_{(0,0,0,0)}\varphi \in (P^r_A(Y))_y$, $y \in Y$. By Lemma 1, we have $w = \mathcal{P}^r_A W_v$ for some projectable-projectable vector field $W \in \mathfrak{X}_{proj\text{-}proj}(Y)$ and $j^r_y W$ is uniquely determined. Then is uniquely determined the $(r-1)$-jet $j^{r-1}_{(0,0,0,0)}((\varphi^{-1})_*W)$, where $(\varphi^{-1})_*W = T\varphi^{-1} \circ W \circ \varphi$. We define

$$\Theta^\lambda_Y(w) := \lambda(j^{r-1}_{(0,0,0,0)}((\varphi^{-1})_*W)). \tag{6}$$

Obviously, $\Theta^\lambda = \{\Theta^\lambda_Y\}$ is $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical V-valued 1-form on P^r_A.

The main result of this note is the following classification theorem.

Theorem 1. *Any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical V-valued 1-form on P^r_A is of the form Θ^λ for some uniquely determined $\mathbb{R}$-linear map*

$$\lambda\colon J^{r-1}_{(0,0,0,0)}(T_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2})) \to V.$$

In the proof of Theorem 1 we use the following fact.

Lemma 2. *Let $W_1, W_2 \in \mathfrak{X}_{proj\text{-}proj}(Y)$ be projectable-projectable vector fields on $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y and let $y \in Y$ be a point. We suppose that $j^{r-1}_y W_1 = j^{r-1}_y W_2$ and $W_1(y)$ is not vertical vector with respect to composition of projections $\pi\colon Y \to X$ and $p\colon X \to M$. Then there exists a (locally defined) $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-map $\Phi\colon Y \to Y$ such that $j^r_y(\Phi) = j^r_y(id_Y)$ and $\Phi_*W_1 = W_2$ near y.*

Proof. It is a direct modification of the proof of Lemma 42.4 in [2]. □

Proof of Theorem 1. Let Θ be $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical V-valued 1-form on P^r_A. We must define $\lambda\colon J^{r-1}_{(0,0,0,0)}(T_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2})) \to V$ by

$$\lambda(\xi) := (\Theta_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(\mathcal{P}^r\widetilde{W}_{j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1,m_2,n_1,n_2}})}) \tag{7}$$

for all $\xi \in J^{r-1}_{(0,0,0,0)}(T_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2}))$, where $\widetilde{W}$ is a unique (germ at $(0,0,0,0)$) of projectable-projectable vector field on $\mathbb{R}^{m_1,m_2,n_1,n_2}$ such that $j^{r-1}_{(0,0,0,0)}\widetilde{W} = \xi$ and coefficients of $\widetilde{W}$ with respect to the basis of space $\mathfrak{X}_{proj\text{-}proj}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ composed of canonical vector fields are polynomials of degree $\leq r-1$. We are going to show that $\Theta = \Theta^\lambda$. Because of the $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-invariance of Θ and Θ^λ it remains to show that

$$(\Theta_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(w) = (\Theta^\lambda_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(w) \tag{8}$$

for any $w \in T_{j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1,m_2,n_1,n_2}})}P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$.

By the definition of λ and Θ^λ we have (8) for any w of the form

$$w = \mathcal{P}^r_A\widetilde{W}_{j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1,m_2,n_1,n_2}})},$$

where $\widetilde{W} \in \mathfrak{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ is a projectable-projectable vector field such that coefficients $\widetilde{W}$ with respect to the above mentioned basis of the space $\mathfrak{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ are polynomials of degree $\leq r-1$.

Now, let $w \in T_{j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1,m_2,n_1,n_2}})} P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$. Then by Lemma 1, w is of the form $w = \mathcal{P}^r_A W_{j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1,m_2,n_1,n_2}})}$ for some projectable-projectable vector field $W \in \mathfrak{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ and $j^r_{(0,0,0,0)}W$ is uniquely determined. We can additionally assume that $W(0,0,0,0)$ is not vertical vector with respect to projection $\mathbb{R}^{m_1+m_2+n_1+n_2} \to \mathbb{R}^{m_1}$. Let $\widetilde{W} \in \mathfrak{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ be projectable-projectable vector field such that $j^{r-1}_{(0,0,0,0)}\widetilde{W} = j^{r-1}_{(0,0,0,0)}W$ and coefficients of field $\widetilde{W}$ with respect to the basis of constant vector fields on $\mathbb{R}^{m_1,m_2,n_1,n_2}$ are polynomials of degree $\leq r-1$. Let $\widetilde{w} = \mathcal{P}^r_A \widetilde{W}_{j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1,m_2,n_1,n_2}})}$. Then (see above) it holds $(\Theta_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(\widetilde{w}) = (\Theta^\lambda_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(\widetilde{w})$.

On the other hand by Lemma 2 there exists a $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-map $\Phi\colon \mathbb{R}^{m_1,m_2,n_1,n_2} \to \mathbb{R}^{m_1,m_2,n_1,n_2}$ such that $j^r_{(0,0,0,0)}\Phi = j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1,m_2,n_1,n_2}})$ and $\Phi_*\widetilde{W} = W$ near $(0,0,0,0) \in \mathbb{R}^{m_1+m_2+n_1+n_2}$. Since $j^r_{(0,0,0,0)}\Phi = id$, then Φ preserves $j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1,m_2,n_1,n_2}})$. Then since $\Phi_*\widetilde{W} = W$, so Φ sends $\widetilde{w}$ into w. Then because of invariance of Θ and Θ^λ with respect to Φ, we obtain

$$\begin{aligned}(\Theta_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(w) &= (\Theta_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(\widetilde{w}) = (\Theta^\lambda_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(\widetilde{w}) \\ &= (\Theta^\lambda_{\mathbb{R}^{m_1,m_2,n_1,n_2}})(w). \qquad \square\end{aligned}$$

For $r=1$ we have $J^0_{(0,0,0,0)}(T_{\textit{proj-proj}}\,\mathbb{R}^{m_1+m_2+n_1+n_2}) \cong \mathbb{R}^{m_1+m_2+n_1+n_2}$. Then by Theorem 1, the vector space of $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical V-valued 1-forms is of dimension $(m_1+m_2+n_1+n_2)\dim V$. Then we have:

Corollary 1. *Any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical* 1*-form $\Theta = \{\Theta_Y\}$ on P^1_A is of the form*

$$\Theta_Y = \lambda \circ \theta_Y \colon TP^1_A(Y) \to V \tag{9}$$

for some unique linear map $\lambda\colon \mathbb{R}^{m_1+m_2+n_1+n_2} \to V$, where $\theta = \{\theta_Y\}$ is a canonical $\mathbb{R}^{m_1+m_2+n_1+n_2}$-valued 1*-form on P^1_A from Example 1.*

Example 3. Notice that it holds

$$J^{r-1}_{(0,0,0,0)}(T_{\textit{proj-proj}}\,\mathbb{R}^{m_1+m_2+n_1+n_2}) \cong \mathbb{R}^{m_1+m_2+n_1+n_2} \oplus \mathfrak{g}^{r-1}_{m_1,m_2,n_1,n_2}\,, \tag{10}$$

where $\mathfrak{g}^{r-1}_{m_1,m_2,n_1,n_2} = \mathcal{L}ie(G^{r-1}_{m_1,m_2,n_1,n_2})$.

In this way for $\lambda = id_{\mathbb{R}^{m_1+m_2+n_1+n_2} \oplus \mathfrak{g}^{r-1}_{m_1,m_2,n_1,n_2}}$ we have $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical 1-form

$$\theta^r_Y := \Theta^{id_{\mathbb{R}^{m_1+m_2+n_1+n_2} \oplus \mathfrak{g}^{r-1}_{m_1,m_2,n_1,n_2}}} : TP^r_A(Y) \to \mathbb{R}^{m_1+m_2+n_1+n_2} \oplus \mathfrak{g}^{r-1}_{m_1,m_2,n_1,n_2} \tag{11}$$

on P^r_A (see Example 2). For $r = 1$, we have $\theta^1 = \theta$ as in Example 1.

Analogously as in Corollary 1 we have

Corollary 2. *Any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-canonical V-valued 1-form $\Theta = \{\Theta_Y\}$ on P^r_A is of the form:*

$$\Theta_Y = \lambda \circ \theta^r_Y \colon TP^r_A(Y) \to V \tag{12}$$

for some uniquely determined linear map $\lambda\colon \mathbb{R}^{m_1+m_2+n_1+n_2} \oplus \mathfrak{g}^{r-1}_{m_1,m_2,n_1,n_2} \to V$, where θ^r is from Example 3.

Remark 1. A notion of fibered square is a generalization of a fibered manifold. The theory of projectable natural bundles over fibered manifolds is essentially related with the idea of fibered square, [2], [3], [5].

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