DOI: 10.2478/v10062-008-0006-2

ANNALES
UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN – POLONIA

VOL. LXII, 2008 SECTIO A 55–60

AGNIESZKA CZARNOTA

Natural affinors on the r-th order adapted frame bundle over fibered-fibered manifolds

Abstract. We describe all $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-natural affinors on the r-th order adapted frame bundle $P^r_A Y$ over (m_1, m_2, n_1, n_2)-dimensional fibered-fibered manifolds Y.

Manifolds and maps are assumed to be of class $\mathbf{C}^\infty$. Manifolds are assumed to be finite dimension and without boundaries.

A fibered-fibered manifold (or a fibered square) is a commutative diagram

$$\begin{array}{ccc} Y & \xrightarrow{\pi} & X \\ {\scriptstyle q}\downarrow & & \downarrow{\scriptstyle p} \\ N & \xrightarrow{\pi_0} & M \end{array} \tag{1}$$

where all maps π, π_0, p, q are surjective submersions and the induced map $Y \to X \times_M N$, $y \mapsto (\pi(y),\ q(y))$ is a surjective submersion, [3], [5]. A fibered-fibered manifold (or fibered square) (1) is denoted by Y for short.

A fibered-fibered manifold (1) has dimension (m_1, m_2, n_1, n_2), if $\dim Y = m_1 + m_2 + n_1 + n_2$, $\dim X = m_1 + m_2$, $\dim N = m_1 + n_1$, $\dim M = m_1$. Recall that for two (m_1, m_2, n_1, n_2)-dimensional fibered-fibered manifolds Y, Y_1, a local isomorphism $f : Y \to Y_1$ is a quadruple of local diffeomorphisms $f : Y \to Y_1$, $f_1 : X \to X_1$, $f_2 : N \to N_1$, $f_0 : M \to M_1$ such that all squares

2000 *Mathematics Subject Classification.* 58A20.

Key words and phrases. Fibered-fibered manifold, r-th order adapted frame bundle, natural affinor.

of corresponding cube are commutative. All fibered-fibered manifolds of dimension (m_1, m_2, n_1, n_2) and their local isomorphisms form a category which we will denote by $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$.

Every $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y is locally isomorphic to the standard fibered-fibered manifold

$$\begin{array}{ccc} \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} & \longrightarrow & \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \\ \downarrow & & \downarrow \\ \mathbb{R}^{m_1} \times \mathbb{R}^{n_1} & \longrightarrow & \mathbb{R}^{m_1} \end{array}$$

which we will denote by $\mathbb{R}^{m_1,m_2,n_1,n_2}$, where arrows are projections.

For any fibered-fibered manifold Y of dimension (m_1, m_2, n_1, n_2) we define the r-th order adapted frame bundle

$$P^r_A Y = \{j^r_{(0,0,0,0)}\varphi \mid \varphi : \mathbb{R}^{m_1,m_2,n_1,n_2} \to Y \text{ is } \mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}\text{-morphism}\} \tag{2}$$

over Y with jet target projection $\beta : P^r_A Y \to Y$, $\beta(j^r_{(0,0,0,0)}\varphi) = \varphi(0,0,0,0)$. Every $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-map $\psi : Y \to Y_1$ induces a map $P^r_A\psi : P^r_A Y \to P^r_A Y_1$ given by $P^r_A\psi(j^r_{(0,0,0,0)}\varphi) = j^r_{(0,0,0,0)}(\psi \circ \varphi)$, [1].

Definition 1. A $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-natural affinor $\mathbf{A}$ on P^r_A is a family of $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-invariant affinors $\mathbf{A} = \{\mathbf{A}_Y\}$ (tensor fields of type $(1,1)$):

$$\mathbf{A}_Y : TP^r_A Y \to TP^r_A Y, \tag{3}$$

on $P^r_A Y$ for any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y, [4].

The invariance means that

$$\mathbf{A}_{Y_1} \circ TP^r_A\psi = TP^r_A\psi \circ \mathbf{A}_Y$$

for any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-map $\psi : Y \to Y_1$.

In this article we describe all $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-natural affinors on P^r_A. All $\mathcal{M}f_m$-natural affinors on P^r were described by Kurek and Mikulski in [4]. We have the following examples of $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-natural affinors on P^r_A.

Example 1. The identity $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-natural affinor *Id* on P^r_A such that $Id : TP^r_A Y \to TP^r_A Y$ is the identity map for any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y.

Remark 1. A vector field W on a fibered-fibered manifold Y is projectable-projectable on $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y, (1), if there exist vector fields W_1 on X and W_2 on N and W_0 on M such that W, W_1 are π-related and W, W_2 are q-related and W_2, W_0 are π_0-related and W_1, W_0 are p-related.

Clearly, a vector field W on a fibered-fibered manifold Y is projectable-projectable on $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y, (1), if and only if the flow $\{\Phi_t\}$ of the vector field W is formed by local $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-maps, [5].

We write $\mathcal{X}_{\textit{proj-proj}}(Y)$ for the space of all projectable-projectable vector fields on $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y. It is a Lie subalgebra of the Lie algebra $\mathcal{X}(Y)$ of all vector fields on Y.

For a projectable-projectable vector field $W \in \mathcal{X}_{\textit{proj-proj}}(Y)$ its flow lifting $\mathcal{P}^r_A W$ is a vector field on $P^r_A(Y)$ such that if $\{\Phi_t\}$ is the flow of W, then $P^r_A(\Phi_t)$ is the flow of $\mathcal{P}^r_A W$.

To give another example of a natural affinor on P^r_A we will use the following lemma, [2].

Lemma 1. *Assume that Y is a fibered-fibered manifold* (1) *of dimension (m_1, m_2, n_1, n_2). Then any vector $w \in T_v P^r_A(Y)$, where $v \in (P^r_A(Y))_y$, $y \in Y$, is of the form $w = \mathcal{P}^r_A W_v$ for some $W \in \mathcal{X}_{\textit{proj-proj}}(Y)$ and $j^r_y W$ is uniquely determined, where $\mathcal{P}^r_A W$ is the flow lifting of W to $P^r_A(Y)$.*

Proof. Clearly, we can assume that $Y = \mathbb{R}^{m_1,m_2,n_1,n_2}$ and $y = (0,0,0,0) \in \mathbb{R}^{m_1+m_2+n_1+n_2}$. Since $P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$ is obviously a principal subbundle of the r-th order frame bundle $P^r(\mathbb{R}^{m_1+m_2+n_1+n_2})$ then by the well-known manifold version of Lemma 1, we find $W \in \mathcal{X}(\mathbb{R}^{m_1+m_2+n_1+n_2})$ such that $w = \mathcal{P}^r W_v$ and $j^r_{(0,0,0,0)} W$ is uniquely determined, where $\mathcal{P}^r W$ is a vector field on $P^r(\mathbb{R}^{m_1+m_2+n_1+n_2})$ being a flow lifting of vector field W and $v \in P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$. For a projectable-projectable vector field $\widetilde{W} \in \mathcal{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ the vector $\mathcal{P}^r \widetilde{W}_v \in T_v P^r(\mathbb{R}^{m_1+m_2+n_1+n_2})$ is tangent to $P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$ at the point v. On the other hand, the dimension of the space $P^r_A(\mathbb{R}^{m_1,m_2,n_1,n_2})$ and the dimension of the space of r-jets $j^r_{(0,0,0,0)} \widetilde{W}$ of projectable-projectable vector fields $\widetilde{W} \in \mathcal{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ are equal. Then Lemma 1 follows from the dimension equality, since the flow operator is linear. □

Example 2. Let

$$B : J^{r-1}_{(0,0,0,0)} T_{\textit{proj-proj}} \mathbb{R}^{m_1,m_2,n_1,n_2} \to \left(J^r_{(0,0,0,0)} T_{\textit{proj-proj}} \mathbb{R}^{m_1,m_2,n_1,n_2} \right)_0$$

be a linear map, where

$$J^{r-1}_{(0,0,0,0)} T_{\textit{proj-proj}} \mathbb{R}^{m_1,m_2,n_1,n_2} = \left\{ j^{r-1}_{(0,0,0,0)} V \mid V \in \mathcal{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2}) \right\}$$

and

$$\left(J^r_{(0,0,0,0)} T_{\textit{proj-proj}} \mathbb{R}^{m_1,m_2,n_1,n_2} \right)_0 = \left\{ j^r_{(0,0,0,0)} V \mid \ V \in \mathcal{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2}), V_{(0,0,0,0)} = 0 \right\}$$

are vector spaces and $\mathcal{X}_{\textit{proj-proj}}(Y)$ is the vector space of all projectable-projectable vector fields on Y. Given a $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y we define a vertical affinor $\mathbf{A}^B_Y : TP^r_A Y \to VP^r_A Y \subset TP^r_A Y$ by

$$\mathbf{A}^B_Y(v) = VP^r_A \varphi((\mathcal{P}^r_A \tilde{v})_\theta), \quad v \in T_{j^r_{(0,0,0,0)}\varphi} P^r_A Y, \quad j^r_{(0,0,0,0)}\varphi \in P^r_A Y, \tag{4}$$

where $v = (\mathcal{P}^r_A \bar{v})_{j^r_{(0,0,0,0)}\varphi}$, $\tilde{v} \in \mathcal{X}_{\textit{proj-proj}}(\mathbb{R}^{m_1,m_2,n_1,n_2})$ is a projectable-projectable vector field on $\mathbb{R}^{m_1,m_2,n_1,n_2}$ with $j^r_{(0,0,0,0)}(\tilde{v}) = B(j^{r-1}_{(0,0,0,0)}(\varphi_\star^{-1}\bar{v}))$ and $\theta = j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1+m_2+n_1+n_2}}) \in P^r_A\mathbb{R}^{m_1,m_2,n_1,n_2}$. Here $\mathcal{P}^r_A V \in \mathcal{X}(P^r_A Y)$ denotes the flow lifting of a projectable-projectable vector field V on Y to $P^r_A Y$. We can show that $\mathbf{A}^B(v)$ is well defined. Precisely $j^r_{\varphi(0,0,0,0)}\bar{v}$ is determined uniquely by v (see Lemma 1).

Then $j^{r-1}_{(0,0,0,0)}(\varphi_\star^{-1}\bar{v}) \in J^{r-1}_{(0,0,0,0)}(T_{\textit{proj-proj}}\mathbb{R}^{m_1,m_2,n_1,n_2})$ is determined uniquely by v and $j^r_{(0,0,0,0)}(\tilde{v}) \in \left(J^r_{(0,0,0,0)}T_{\textit{proj-proj}}\mathbb{R}^{m_1,m_2,n_1,n_2}\right)_0$ is determined by v. Then $(\mathcal{P}^r\tilde{v})_\theta$ is determined by v and it is a vertical vector. Thus $\mathbf{A}^B_Y(v)$ is determined by v and it is a vertical vector. Using the linearity of the flow operator we obtain that $\mathbf{A}^B_Y : TP^r_A Y \to VP^r_A Y \subset TP^r_A Y$ is a vertical affinor.

It is easy to see that the family $\mathbf{A}^B = \{\mathbf{A}^B_Y\}$ of affinors $\mathbf{A}^B_Y : TP^r_A Y \to TP^r_A Y$ for any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y is a $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-natural affinor on P^r_A.

The main result of the present note is the following classification theorem:

Theorem 1. *Any $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-natural affinor $\boldsymbol{A}$ on P^r_A is of the form*

$$\boldsymbol{A} = \lambda Id + \boldsymbol{A}^B, \tag{5}$$

for a (uniquely determined by $\boldsymbol{A}$) real number λ and a (uniquely determined by $\boldsymbol{A}$) linear map

$$B : J^{r-1}_{(0,0,0,0)}T_{\textit{proj-proj}}\mathbb{R}^{m_1,m_2,n_1,n_2} \to \left(J^r_{(0,0,0,0)}T_{\textit{proj-proj}}\mathbb{R}^{m_1,m_2,n_1,n_2}\right)_0. \tag{6}$$

In the proof of Theorem 1 we use the following fact.

Lemma 2. *Let $W_1,\ W_2 \in \mathcal{X}_{\textit{proj-proj}}(Y)$ be projectable-projectable vector fields on an $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-object Y and let $y \in Y$. Let us assume that $j^{r-1}_y W_1 = j^{r-1}_y W_2$ and $W_1(y)$ is not vertical with respect to the composition of the projections $\pi : Y \to X$ and $p : X \to M$. Then there exists a local $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-morphism $\Phi : Y \to Y$ such that $j^r_y(\Phi) = j^r_y(id_Y)$ and $\Phi_\star W_1 = W_2$ near y.*

Proof. The proof is a simple modification of the proof of Lemma 42.4 in [2]. □

Proof of Theorem 1. Let $\theta = j^r_{(0,0,0,0)}(id_{\mathbb{R}^{m_1+m_2+n_1+n_2}}) \in P^r_A\mathbb{R}^{m_1,m_2,n_1,n_2}$. Suppose that $\mathbf{A}((\mathcal{P}^r_A V)_\theta) = (\mathcal{P}^r_A\tilde{V})_\theta$ and $V(0,0,0,0) \neq \mu\tilde{V}(0,0,0,0)$ for all μ and $\tilde{V}(0,0,0,0) \neq 0$. Then there exists an $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-map $\psi : \mathbb{R}^{m_1,m_2,n_1,n_2} \to \mathbb{R}^{m_1,m_2,n_1,n_2}$ preserving θ with $J^rT\psi(j^r_{(0,0,0,0)}V) = j^r_{(0,0,0,0)}V$ and $J^rT\psi(j^r_{(0,0,0,0)}\tilde{V}) \neq j^r_{(0,0,0,0)}\tilde{V}$. Then

$$\mathbf{A}((\mathcal{P}^r_A V)_\theta) = (\mathcal{P}^r_A(\psi_\star\tilde{V}))_\theta \neq (\mathcal{P}^r_A\tilde{V})_\theta = \mathbf{A}((\mathcal{P}^r_A V)_\theta) \tag{7}$$

and it is a contradiction. Then

$$T\beta^r \circ \mathbf{A}((\mathcal{P}^r_A V)_\theta) = \lambda(j^r_{(0,0,0,0)}V)V_{(0,0,0,0)}, \tag{8}$$

for some (not necessarily unique and necessarily smooth) function

$$\lambda : J^r_{(0,0,0,0)}T_{\textit{proj-proj}}\,\mathbb{R}^{m_1,m_2,n_1,n_2} \to \mathbb{R},$$

where $\beta^r : P^r_A\mathbb{R}^{m_1,m_2,n_1,n_2} \to \mathbb{R}^{m_1+m_2+n_1+n_2}$ is the usual projection.

We prove that λ can be chosen from smooth functions. Let λ be such that (8) holds. Since the left side of (8) depends smoothly on $j^r_{(0,0,0,0)}V$ then the function $\Phi : J^r_{(0,0,0,0)}(T_{\textit{proj-proj}}\,\mathbb{R}^{m_1,m_2,n_1,n_2}) \to \mathbb{R}$ given by

$$\Phi(j^r_{(0,0,0,0)}V) = \lambda(j^r_{(0,0,0,0)}V)V^1(0)\ , \qquad 0 \in \mathbb{R}^{m_1,m_2,n_1,n_2} \tag{9}$$

is smooth and $\Phi(j^r_{(0,0,0,0)}V) = 0$ if $V^1(0) = 0$ where

$$V_{(0,0,0,0)} = \sum_{i=1}^{m_1} V^i(0)\frac{\partial}{\partial x^i}|_{(0,0,0,0)} + \dots. \tag{10}$$

Then (it is the well-known fact from mathematical analysis) there is a smooth map $\psi : J^r_{(0,0,0,0)}(T_{\textit{proj-proj}}\,\mathbb{R}^{m_1,m_2,n_1,n_2}) \to \mathbb{R}$ such that

$$\Phi(j^r_{(0,0,0,0)}V) = \psi(j^r_{(0,0,0,0)}V)V^1(0). \tag{11}$$

Then we can put $\lambda = \psi$ and (8) holds.

Since the left hand side of (8) depends linearly on $j^r_{(0,0,0,0)}V$ we have $\lambda = \textit{const}$. Replacing $\mathbf{A}$ by $\mathbf{A} - \lambda Id$ we see that $\mathbf{A}(v)$ is vertical for any $v \in T_\theta P^r_A\mathbb{R}^{m_1,m_2,n_1,n_2}$.

We define a linear map

$$B : J^{r-1}_{(0,0,0,0)}T_{\textit{proj-proj}}\,\mathbb{R}^{m_1,m_2,n_1,n_2} \to \left(J^r_{(0,0,0,0)}T_{\textit{proj-proj}}\,\mathbb{R}^{m_1,m_2,n_1,n_2}\right)_0 \tag{12}$$

by

$$B(j^{r-1}_{(0,0,0,0)}V) = j^r_{(0,0,0,0)}\tilde{V}, \tag{13}$$

where $\bar{V}$ is a unique projectable-projectable vector field on $\mathbb{R}^{m_1,m_2,n_1,n_2}$ with coefficients being polynomials of degree $\leq r-1$ (with respect to the canonical basis of vector field on $\mathbb{R}^{m_1,m_2,n_1,n_2}$) such that $j^{r-1}_{(0,0,0,0,)}\bar{V} = j^{r-1}_{(0,0,0,0)}V$, and $\mathcal{P}^r_A(\tilde{V})_\theta = \mathbf{A}((\mathcal{P}^r_A\bar{V})_\theta)$.

We will show that $\mathbf{A} = \mathbf{A}^B$. Clearly B is well defined. (For, $j^r_{(0,0,0,0)}\tilde{V}$ is determined by $(\mathcal{P}^r_A\tilde{V})_\theta = \mathbf{A}((\mathcal{P}^r_A\bar{V})_\theta)$ and $\mathcal{P}^r_A(\bar{V})_\theta$ is determined by $j^r_{(0,0,0,0)}\bar{V}$ (see Lemma 1) and $j^r_{(0,0,0,0)}\bar{V}$ is determined by $j^{r-1}_{(0,0,0,0)}V$ (by the definition of $\bar{V}$)). Moreover, since $\mathbf{A}$ is of vertical type then $\tilde{V}(0,0,0,0) = 0$. That is why B is well defined. Then (by the definition of B) we see that $\mathbf{A}((\mathcal{P}^r_A V)_\theta) = \mathbf{A}^B((\mathcal{P}^r_A V)_\theta)$ for all projectable-projectable vector fields V on $\mathbb{R}^{m_1,m_2,n_1,n_2}$ with coefficients being polynomials of degree $\leq r-1$ (with respect to the canonical basis of vector fields on $\mathbb{R}^{m_1+m_2+n_1+n_2}$). But (by Lemma 2) any projectable-projectable vector fields W on $\mathbb{R}^{m_1,m_2,n_1,n_2}$ with non-vanishing projection on $\mathbb{R}^{m_1}$ is ψ-related (near $(0,0,0,0)$) to some projectable-projectable vector field V with coefficients being polynomials of degree $\leq r-1$ for some θ-preserving $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-map $\psi : \mathbb{R}^{m_1,m_2,n_1,n_2} \to \mathbb{R}^{m_1,m_2,n_1,n_2}$. Consequently $\mathbf{A}(v) = \mathbf{A}^B(v)$ for any $v \in T_\theta P^r_A \mathbb{R}^{m_1,m_2,n_1,n_2}$. Then $\mathbf{A} = \mathbf{A}^B$ because of the $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-invariance of $\mathbf{A}$ and $\mathbf{A}^B$ and the fact that $P^r_A Y$ is the $\mathcal{F}^2\mathcal{M}_{m_1,m_2,n_1,n_2}$-orbit of θ.

If $\mathbf{A}^B = \mathbf{A}^{B'}$ then $B = B'$. If λ' i B' are such that $\mathbf{A} = \lambda' Id + \mathbf{A}^{B'}$, then $\lambda = T\beta^r \circ \mathbf{A}((\mathcal{P}^r_A \frac{\partial}{\partial x^1})_\theta) = \lambda'$ and $B = B'$. $\square$

Remark 2. Natural affinors on $P^r_A Y$ can be used to define a generalized torsion of connections on $P^r_A Y$. Any natural affinor $\mathbf{A} : TP^r_A Y \to TP^r_A Y$ defines a torsion $\tau^A(\Gamma) := [\mathbf{A}, \Gamma]$ of a principal r-th order connection $\Gamma : TP^r_A Y \to TP^r_A Y$ on fibered-fibered manifold Y, where the bracket means the Frölicher–Nijenhuis bracket.

A principal r-th order connection Γ on $P^r_A Y \to Y$ is a right invariant section $\Gamma : P^r_A Y \to J^1 P^r_A Y$ of the first jet prolongation $J^1 P^r_A Y \to P^r_A Y$ of $P^r_A Y \to Y$. Equivalently, Γ can be treated as the corresponding lifting map $\Gamma : TY \times_Y P^r_A Y \to TP^r_A Y$, [2].

References

[1] Doupovec, M., Mikulski, W. M., *Gauge natural constructions on higher order principal prolongations*, Ann. Polon. Math. **92** (2007), no. 1, 87–97.

[2] Kolář, I., Michor, P. W. and Slovák, J., *Natural Operations in Differential Geometry*, Springer-Verlag, Berlin, 1993.

[3] Kolář, I., *Connections on fibered squares*, Ann. Univ. Mariae Curie-Skłodowska Sect. A **59** (2005), 67–76.

[4] Kurek, J., Mikulski, W. M., *The natural affinors on the r-th order frame bundle*, Demonstratio Math. **41** (2008), 701–704.

[5] Mikulski, W. M., *The jet prolongations of fibered-fibered manifolds and the flow operator*, Publ. Math. Debrecen **59 (3-4)** (2001), 441–458.

Agnieszka Czarnota
Institute of Mathematics
M. Curie-Skłodowska University
pl. Marii Curie-Skłodowskiej 1
20-031 Lublin, Poland
e-mail: czarnota.agnieszka@interia.pl

Received June 12, 2008